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IIW Guideline for the Assessment of Weld Root Fatigue

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Abstract

At welded joints, fatigue cracks usually initiate either at the weld toe or the weld root. The latter may be influenced by small defects or other irregularities or even by non-fused root faces forming a slit which acts like a crack. Weld root failure has therefore to be checked for several weld types. Different approaches exist for fatigue assessment, including the rather simple nominal stress approach, local stress approaches and the crack propagation approach which are partly well-suited for the assessment of weld root fatigue. However, it is not easy to keep the overview and to decide which approach should be applied in the case in question. For this purpose, the guideline has been established giving an overview of the different approaches with special emphasis on weld roots. In addition, six typical examples are described where different approaches are applied and in some cases compared with fatigue tests, thus giving insight into the practical application and allowing own judgement.

Keywords: Fatigue, weld root, fatigue assessment, nominal stress, structural stress, notch stress, crack propagation, FEM

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1 Introduction

It is well-known that welded structures being subjected to cyclic loading are prone to fatigue. Fatigue cracks initiate at the locations with high stress concentration. These are the relatively sharp transitions between the parent and weld metal at the weld toes as well as weld roots which are frequently characterized by sharp slits with non-fused root faces acting like crack-starters.

The situation is often illustrated by the example of a cruciform joint with load-carrying fillet welds, see Fig. 1.1. Alternatively to cracks initiating from the weld toes, root cracks may start from the ends of the non-welded root faces, separating the weld throat during crack growth. The question, which crack type dominates, depends mainly on the ratio between throat and plate thickness (including possible penetration), but also on the type of loading, weld shape, misalignment, residual stresses etc.

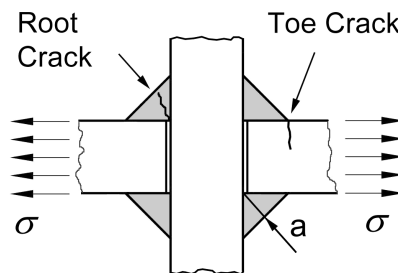


Fig. 1.1: Possible crack locations in a cruciform joint with load-carrying fillet welds

Toe and root cracks can be distinguished in failure cases by their characteristic appearance. While toe cracks are located in the weld toe line - at least in the beginning -, weld root cracks appear on the weld surface after penetrating through the weld material. Fig. 1.2 shows examples for a weld toe and root crack at the fillet weld around a stiffener termination.

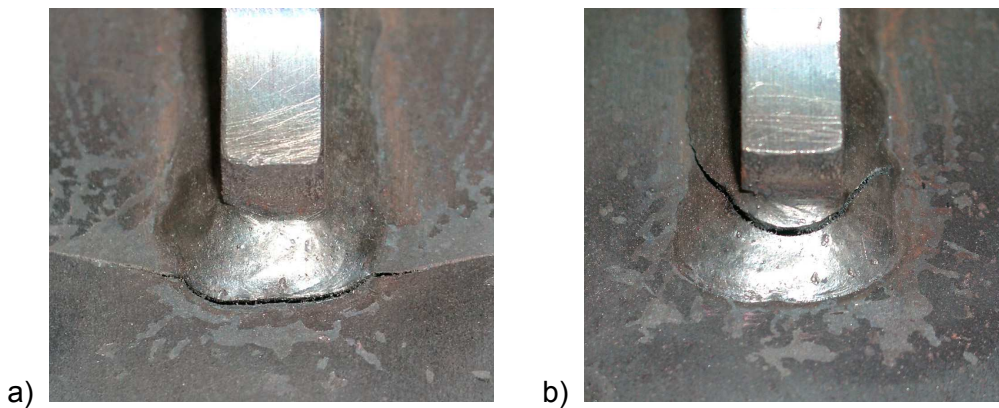


Fig. 1.2: Weld toe (a) and root crack (b) at the fillet weld around a stiffener termination

Weld root cracks are frequently considered to be more dangerous than weld toe cracks as their initiation cannot be detected before they have grown through the whole weld thickness. On the other hand, the residual life can still be large particularly in cases with stress concentration and the continuous base plate may even remain intact. Nevertheless, an adequate fatigue strength assessment is necessary to ensure the integrity of the structure and to avoid early and unexpected fatigue failures. This is particularly necessary for optimised structures having welds with minimized dimensions or post-weld treatment. The latter can improve only the weld toe and not the weld root.

Various approaches for fatigue strength assessment are available (Hobbacher, 2009), which are not only applicable to weld toe, but also to weld root failure. The approaches differ mainly in the stress parameter used for the assessment:

- *nominal stress approach*, based on the stress disregarding any stress increase due to the structural detail or the weld; in the case of weld root failure, a nominal stress based on the weld throat thickness has to be used
- *structural stress approach*, based on the stress containing only the stress increase due to the structural geometry, but not due to the local weld geometry; the approach has been developed for weld toe failures, but has been extended to some cases with weld root fatigue at fillet welds, using a linearized stress in the weld
- *effective notch stress approach*, based on the local stress at the rounded weld toe or weld root notch, assuming ideal-elastic material behaviour and microstructural support effects to a certain extent
- approaches based on the *stress intensity factor*, using the *notch stress intensity factor* (N-SIF) of the weld toe with zero radius as fatigue parameter. For the weld root, the stress intensity factor for crack tips is used
- *crack propagation approach*, using Paris law for determining the fatigue life of a propagating crack; while the actual non-fused part is considered as initial crack at the weld root, an initial crack depth must be assumed for the weld toe

Before the weld root assessment is described for the different approaches, the scope of applications covered by this guideline is outlined at first. Finally, several demonstration examples are described, illustrating the range and details of practical application.

Reference is made to several papers, recommendations and guidelines giving more details of the different approaches. It is recommended to apply different approaches to the actual problem in order to verify the results.

2 Scope of Applications

2.1 Butt Joints

Typical butt joints with non-fused root faces at risk of weld root fatigue are shown in Fig. 2.1. In type a), a two-sided butt weld with embedded root faces, the load-carrying section is reduced to $2 \cdot a$. This case is contained in several detail catalogues of fatigue codes and guidelines (e. g. detail 217 in Hobbacher, 2009), where the stress has to be based on the throat sectional area ($2 \cdot a$) and a very low fatigue class is assumed. This leads to low fatigue strength so that this detail is not recommended for cyclic loaded structural members.

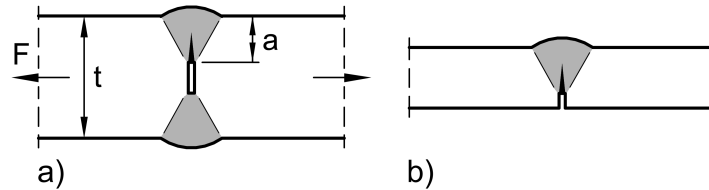


Fig. 2.1: Butt joints with non-fused root faces at risk of weld root fatigue (cracks indicated)

Non-fused root faces also occur in one-sided butt welds made without backing bar (i. e. no permanent nor ceramic type). In this case, the non-fused root faces can result in early cracks starting from the root. Secondary bending can worsen the situation. Also this case is a typical one in codes and guidelines (e. g. detail 216 in Hobbacher, 2009), where a very low fatigue class is assumed particularly in cases where the weld root cannot be checked by appropriate non-destructive testing.

If, however, the weld root is checked by appropriate NDT and contains rather small axial misalignment, the fatigue strength can be higher (Maddox and Razmjoo, 1998). This is utilized in the design of pipelines by defining appropriate measures (NDT, low misalignment) and allowing fatigue classes up to FAT 56 - 63 (DNV, 2010) or even 71 (detail 216 acc. to Hobbacher, 2009).

2.2 Fillet Welds

Fig. 2.2 shows different types of cruciform joints with non-fused root faces. Types a) and b) are double bevel butt welds and two-side fillet welds without penetration. Here, the throat thickness a determines the weld stress and thus the risk of root fatigue. The depth e of the penetration (excluding possible root defects) increases the load-carrying section and hence the fatigue strength. Corresponding fatigue classes can be found in codes and guidelines (e. g. detail 414 in Hobbacher, 2009).

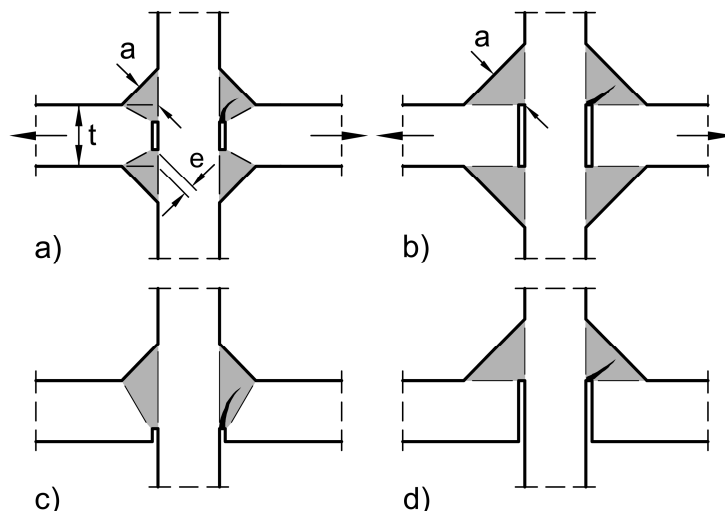


Fig. 2.2: Fillet welds with non-fused root faces at risk of weld root fatigue (cracks indicated)

The necessary weld throat thickness a or leg length c to avoid fractures from weld root can be determined from the fatigue classes based on fatigue tests of from fatigue assessment procedures described in the following section. A typical example for cruciform steel joints with partial penetration welds under axial loading is given in Fig. 2.3 showing limit curves separating geometries leading to either fatigue fractures from the weld toe and root, respectively, derived from fatigue tests. An increased leg length ratio c/b results avoids weld root failures.

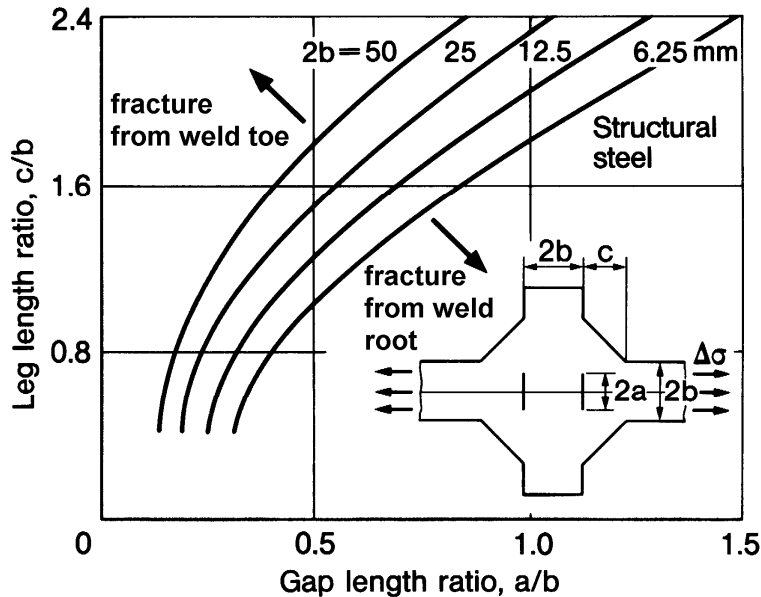


Fig. 2.3: Limit curves separating fracture from weld toe and root in cruciform joints (Maddox, 1974)

The one-sided welds (types c) and d) in Fig. 2.2) are less fatigue-resistant because the weld throat is subjected to additional bending in the case of axially-loaded plates. The bending stress is considered in corresponding fatigue classes (e. g. detail 416 in Hobbacher, 2009). Usually such welds are only acceptable if weld bending is kept under control e. g. by transverse stiffening plates. Also circular or small rectangular hollow sections can provide sufficient restraint, allowing fatigue classes to be based on nominal axial stress only (e. g. details 423 and 425 in Hobbacher, 2009).

The welded joints are frequently subjected to loads varying along the weld line which may be due to the geometrical configuration. A typical case is the fillet-welded end of a loaded stiffener where a stress concentration normally occurs and the non-fused root faces terminate (Fig. 2.4). This requires a local stress analysis and corresponding fatigue assessment.

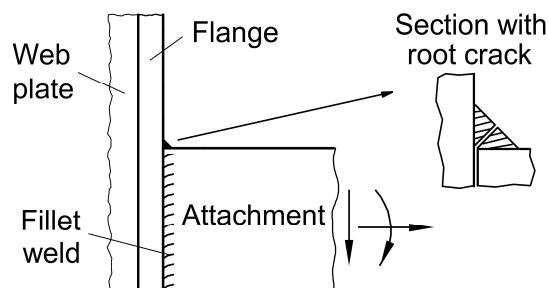


Fig. 2.4: Fillet-welded end of a loaded stiffener

2.3 Other Welds

Special welding processes create connections with limited penetration so that non-welded slits occur. Examples are spot-welds, laser stake-welds and one-sided laser welds e. g. at T-joints, see

Fig. 2.5. Fatigue cracks may occur at the ends of the slits, separating either the weld or parent metal which depends on the loading condition.

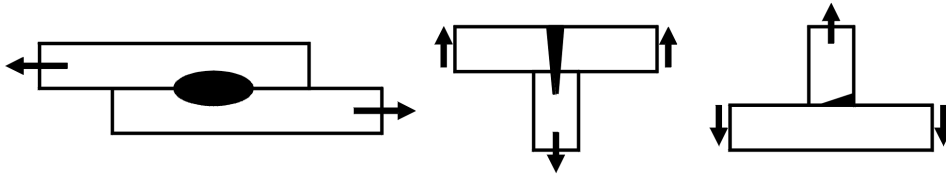


Fig. 2.5: Other welds susceptible for weld root fatigue

The situation is similar to but- and fillet-welds with partial penetration so that the same approaches for fatigue assessment can be applied. One major difference is the loading which is frequently dominated by shear forces in the connected planes. Thus, special internal forces and moments are defined in some cases resulting in specific nominal and/or structural stresses (Radaj et al., 2006).

3 Procedures for Fatigue Assessment

3.1 General Aspects

3.1.1 General Procedures

The fatigue assessment procedure can be based on

- S-N curve showing the relationship between the fatigue life in terms of constant amplitude load cycles and a stress parameter, which may be the range of a nominal or local stress in the weld or a stress intensity at a notch. Variable amplitude stresses are taken into account using a damage accumulation hypothesis, normally the linear Palmgren-Miner rule (or Miner's rule).
- crack growth curve established with a crack propagation law such as the Paris-Erdogan law, based on fracture mechanics and assuming an initial crack length and appropriate material parameters for the analysis. Variable load amplitudes can be considered by an incremental analysis using single load cycles or blocks with different load amplitudes.

Further information is given e. g. by Hobbacher (2009) and by several textbooks on fatigue. The following sub-chapters outline the approaches using different stress parameters as well as the approach for crack propagation. The approaches are generally affected by several influence parameters, some of which being relevant for root cracking are generally discussed below.

3.1.2 Residual Stress

It is well known that residual stresses significantly influence the fatigue behaviour (Sonsino, 2009a). High tensile residual stresses reduce the fatigue life, while compressive residual stresses increase the fatigue life as long as the load ratio is not too high. Crack initiation as well as crack propagation is influenced, the latter mainly by crack closure.

At weld toes it is usually assumed that unfavourable residual stresses due to welding are present which may reach the yield stress of the material. Therefore, test results for a high stress ratio ($R = 0.5$) are conservatively assumed for any loading (Hobbacher, 2009), however, weld toes at stiffener ends (Fig. 2.3) may also show compressive residual stresses due to welding (Fricke, 2005).

Welding residual stresses are difficult to measure at the weld root. However, simulations of fillet and butt welds have shown compressive residual stresses at weld root at a magnitude of the yield strength of the base material in mild strength steels (Barsoum, 2009) for multi-pass welds. Barsoum (2008) also showed that the residual stresses at root side depend on weld penetration and the number of weld passes, where an increase of the compressive residual stresses were observed with increased weld penetration and the number of weld passes. However, the final residual stress at the weld root side for multi-pass welding also depends on the inter-pass time.

It has been observed that the crack initiation points at pipe-to-pipe connections depend highly on the residual stress state being affected by the welding sequence.

As an unfavourable global residual stress state may be present due to the fabrication and erection of the whole structure, high residual stresses should be assumed also for the weld root unless more favourable conditions are proven.

3.1.3 Misalignment

Another factor affecting fatigue strength is misalignment creating secondary bending stresses which have to be considered in all local approaches including the crack propagation approach.

Misalignment effects are particularly large in welds at plates subjected to loading transverse to the weld. For instance, the axial misalignment e of an unrestrained cruciform joint (Fig. 3.1) creates at the weld toe an increase of the axial stress, which can be expressed by the following stress magnification factor k_m :

$$k_m = 1 + 3 \cdot e/t \quad (3.1)$$

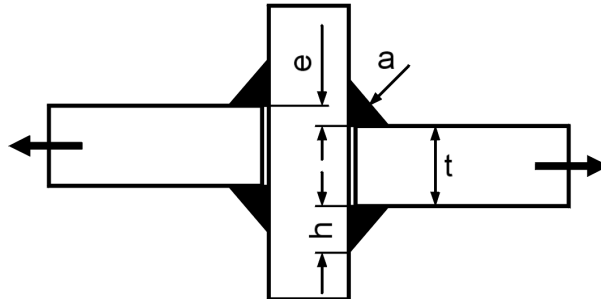


Fig. 3.1: Axial misalignment e of a cruciform joint

The secondary bending stress is considerably reduced at the weld root, which has been investigated by Andrews (1988). He derived the following formula for the stress magnification at the weld root:

$$k_m = 1 + e/(t + h) = 1 + e/(t + a \sqrt{2}) \quad (3.2)$$

The right part applies to fillet welds with 45° flank angle. Other cases may be analysed by simple beam theory. Additional restraint in the weld direction, e. g. by stiffeners or shell effects, may require a 3D finite element analysis.

3.2 Nominal Stress Approach

3.2.1 Overview of the approach

The nominal stress approach uses for the fatigue assessment a stress, which disregards the local stress increase due to the structural discontinuity (e. g. a stiffener termination) and also due to the local weld profile. These effects are included in notch cases which are associated with S-N curves allowing the fatigue life to be assessed (FAT classes).

In the case of weld root fatigue, a nominal stress in the weld is used. This might require a special analysis which is outlined below. As already mentioned in chapter 2, several notch cases are defined in codes and guidelines for different types of joints (e. g. Hobbacher, 2009). The associated fatigue class is relatively low (FAT 36 - FAT 40), defining the characteristic fatigue strength of the design S-N curve at 2 million cycles for a survival probability of 97.7%. The reasons for the low fatigue class are that a very sharp notch is present and that the nominal stress in the weld disregards any local bending effects.

3.2.2 Nominal weld stress

The nominal stress is often determined by formulas using load parameters such as forces and bending moments as well as section properties such as section area and modulus. Alternatively, relatively coarse finite element models are used. It should be kept in mind that stress raisers, which are not typical for the welded detail itself, have to be additionally taken into account (e. g. reduced effective widths).

In *butt joints*, the nominal weld stress can often be calculated directly from the nominal stress in the adjacent plate taking into account any reduced sectional area, see Fig. 2.1. Weld reinforcement is usually neglected.

In *fillet welds*, a special weld stress is usually applied which is illustrated in Fig. 3.2a by a cruciform joint with fillet welds. The weld stress $\sigma_{n,w}$ is based on averaged stress components in the weld throat which are also used in the static design of welds, i. e.

- the stress $\sigma_{\perp}$ normal to the weld throat section
- the shear stress $\tau_{\perp}$ in the weld throat section

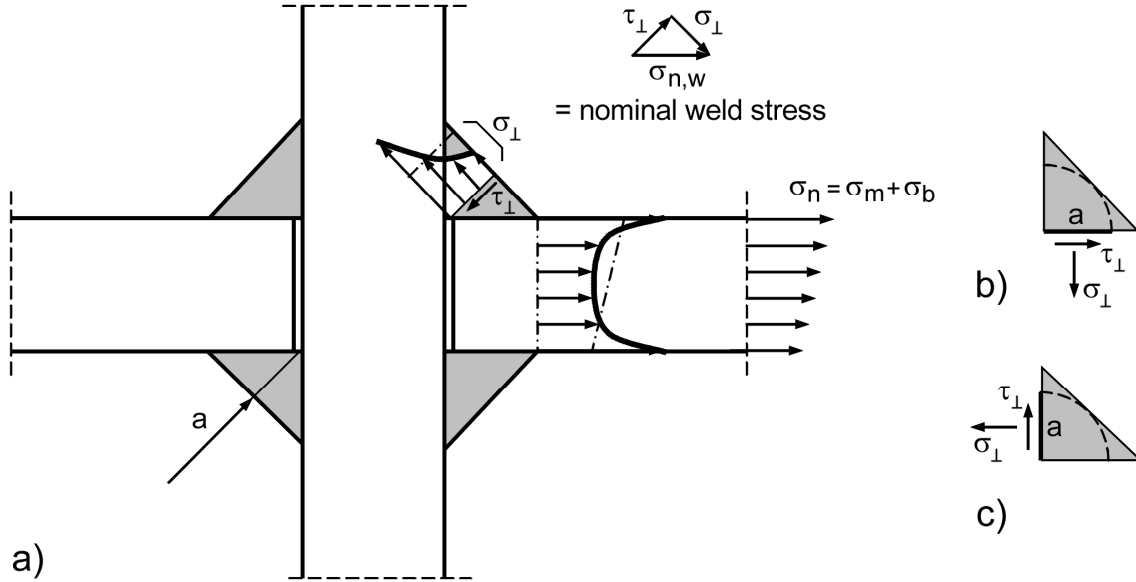


Fig. 3.2: Fillet-welded cruciform joint with nominal stresses in plate and weld

As illustrated in Fig. 3.2b and c, also the horizontal or vertical leg sections, reduced to the throat thickness a , may be considered to determine the averaged stress components $\sigma_{\perp}$ and $\tau_{\perp}$.

The stress components are combined to the nominal weld stress $\sigma_{n,w}$ by vector addition, i. e.

$$\sigma_{n,w} = \sqrt{\sigma_{\perp}^2 + \tau_{\perp}^2} \quad (3.3)$$

In simple cases, the stress components can be determined by equilibrium conditions from the nominal stress σ_n in the plate or in more complex cases by the finite element method using the forces or stresses in the weld.

In the case of the cruciform joint, the nominal weld stress can also be determined by simply referring the axial force in the plate ($\sigma_m \cdot t$) to the weld throat area ($2a$) and the bending moment ($\sigma_b \cdot t^2/6$) to the weld throat area multiplied by the distance between the weld legs ($a \cdot t$):

$$\sigma_{n,w} = \sigma_m \frac{t}{2a} + \sigma_b \frac{t}{6a} \quad (3.4)$$

Other cases can be treated analogously. Special cases are one-sided fillet and partial penetration Y-butt welds, where the secondary bending stress is casually considered in the nominal weld stress in connection with an increased fatigue class. The stress is frequently regarded as a structural stress, see Section 3.3.

3.2.3 Notch cases

Codes and guidelines based on the nominal stress approach usually contain several notch cases for weld root fatigue. Chapter 2 summarizes the cases contained in the IIW Fatigue Recommendations (Hobbacher, 2009) without stating all details.

The fatigue classes may be grouped according to the following principles:

- *Butt joints* made from *one side with weld root controlled* by a backing bar or non-destructive testing (NDT) are classified as FAT 71 - 80 for steel and FAT 25 - 28 for aluminium.
- If *butt joints* contain *embedded non-fused root faces* and/or if the *weld root cannot be controlled* by NDT, the joint is classified as FAT 36 (steel) or FAT 12 (aluminium).
- *Fillet welds and partial penetration K-butt welds* are classified as FAT 36 (steel) or FAT 12 (aluminium). Relatively thin fillet welds ($a/t \leq 1/3$) are assessed with one higher class. Exceptions are one-sided fillet and partial penetration Y-butt welds, if the secondary bending stress is included (leading to FAT 71 for steel and FAT 25 for aluminium). Welds connecting the end of a hollow section lead to FAT 36 - 45 for steel and FAT 14 - 16 for aluminium (depending of the section shape and thickness)
- Special cases are *butt-welds of three-plate joints* and *fillet welds at lap joints*. Here, the non-fused root faces are parallel to the loading direction. In the case of butt-welds, FAT 63 - 71 applies for steel and FAT 22 for aluminium. Double lap joints are classified as FAT 45 for steel and FAT 16 for aluminium, whereas overlap joints are downgraded to FAT 36 (steel) and FAT 12 (aluminium) due to the additional throat bending.

3.2.4 Multiaxial loading

Further stress components may be acting in addition to those shown in Fig. 3.2. Relevant for fatigue is mainly the shear stress $\tau_{||}$ parallel to the weld line. Interaction equation according to Gough-Pollard, which corresponds to the von Mises hypothesis, can be used describing the combined effect of stress components, i. e. $\sigma_{n,w}$ and $\tau_{||}$ in this case:

$$\left(\frac{\sigma_{n,w}}{FAT(\sigma, N)} \right)^2 + \left(\frac{\tau_{||}}{FAT(\tau, N)} \right)^2 \leq D_{MA} \quad (3.5)$$

with the multiaxial damage parameter $D_{MA} = 1$ and the particular fatigue strengths (FAT) for normal and shear stresses at the anticipated number of load cycles N , see Sonsino and Wiebesiek (2007), Hobbacher (2009), Wiebesiek and Sonsino (2010), Sonsino (2009c) and Sonsino (2011).

If the normal stress component parallel to the weld $\sigma_{||}$ is to be considered too, then this has to be done by deriving either the principal stress or the von Mises stress from $\sigma_{n,w}$ and $\sigma_{||}$, but without the shear stress component $\tau_{\perp}$.

A further alternative for the case of proportional stress components, an extension of eq. (3.3), has been proposed (Lotsberg, 2003), using the averaged parallel shear stress in the weld throat $\tau_{||}$:

$$\sigma_{n,w} = \sqrt{\sigma_{\perp}^2 + \tau_{\perp}^2 + 0.2 \cdot \tau_{||}^2} \quad (3.6)$$

The above proposed equation (3.5) results acceptable evaluations for proportional loading with constant principal stress directions. However, in case of non-proportional loading with changing principal stress directions these equations, including (3.3), fail, because they do not display the fatigue life reduction for ductile materials and overestimate fatigue life significantly, i.e. the calculations are on the unsafe side. For this, a modification of the Gough-Pollard equation (3.5) by the multiaxial damage parameter $D_{MA} = 0.5$ is suggested, rendering good assessments see Sonsino and Wiebesiek (2007), Wiebesiek and Sonsino (2010), Sonsino (2009c) and Sonsino (2011).

3.3 Structural Stress Approaches

3.3.1 Overview of the approaches

In contrast to the nominal stress, the *structural stress* contains the stress increase due to the structural configuration, e. g. caused by the *structure-related stress concentration* or additional *plate bending* including significant misalignment effects. Not considered is the local stress peak by the weld itself due to its sharp notches at the weld toe and root.

At *weld toes*, the structural stress is usually determined by extrapolating the surface stress to the hot-spot, being termed *structural hot-spot stress* (Niemi et al., 2006). Another way of omitting the local stress peak is the stress linearization in the thickness direction.

Surface stress extrapolation is not suitable for the fatigue assessment of weld roots. Therefore, approaches exist which utilize a *local nominal stress* or a *structural weld stress* derived from the forces and moments or the stress distribution in the weld itself, partly restricted to definite cases of application. Fig. 3.3 gives an overview of the different approaches together with the stress types used and the FAT classes at steel. The approaches are briefly described in the following.

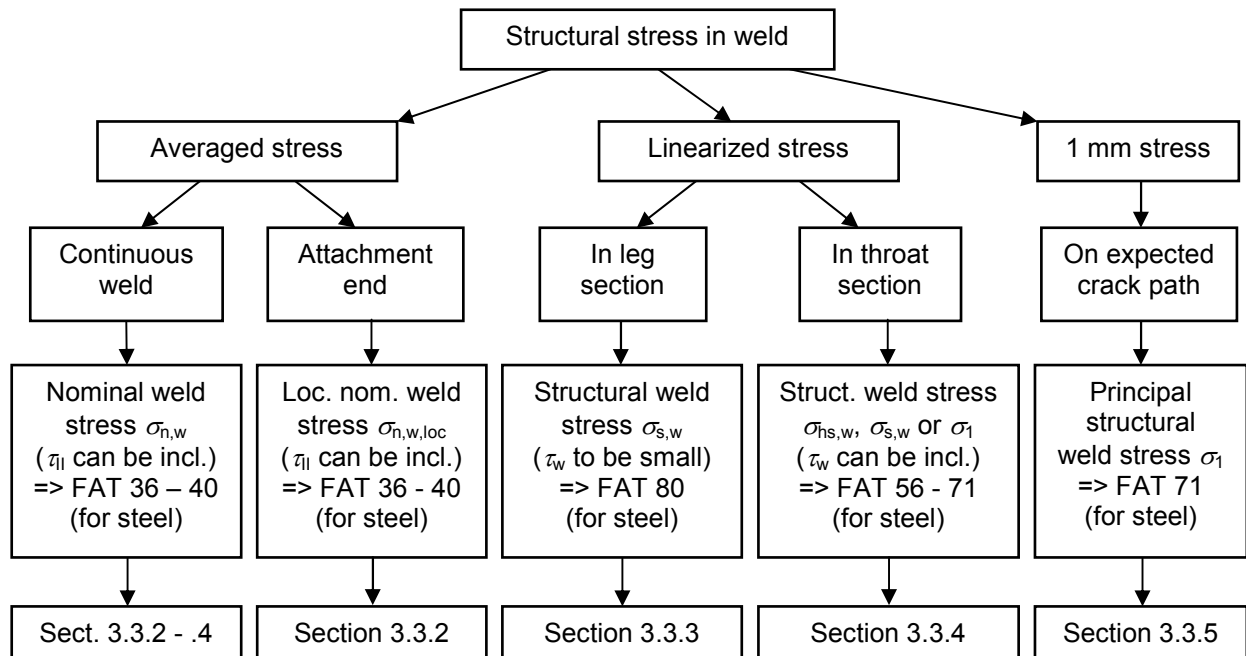


Fig. 3.3: Overview of the different structural stress approaches for weld root fatigue assessment

3.3.2 Approach for fillet-welded attachment ends

In fillet welds around loaded attachment ends, as illustrated in Fig. 2.4, increased stresses occur due to the concentrated force flow in the corner. The increased stress averaged in the weld throat area around the attachment end can be estimated from the transmitted force F_e as outlined in Fig. 3.4 (Fricke and Doerk, 2006), while the local weld throat bending is neglected. This stress can be considered as *structural weld stress* or *local nominal weld stress*, averaged over a definite area.

It has to be distinguished between two cases. In the *first case*, the plate stress σ_p in the attachment is acting perpendicular to the fillet weld (Fig. 3.4) without any other significant stress component such as a shear stress parallel to the weld. Weld root failure is caused by the stress transferred through the fillet weld having the throat thickness a .

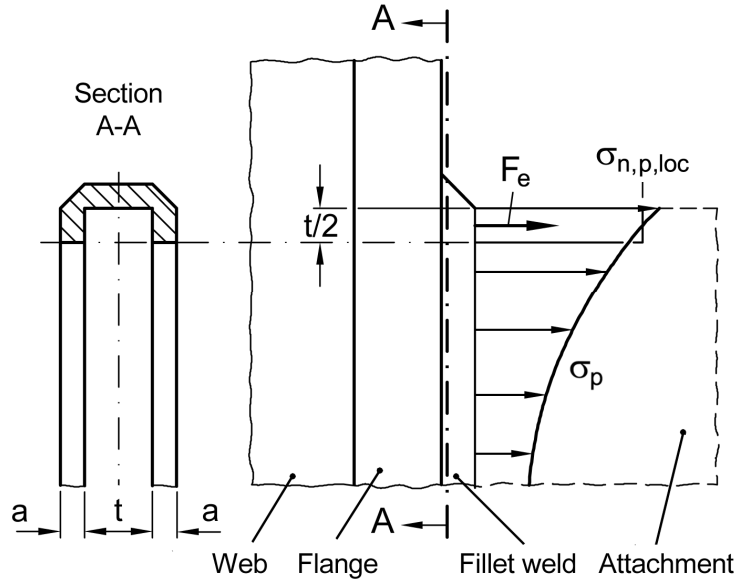


Fig. 3.4: Geometry and loading of the end of an attachment being similar to that in Fig. 2.3

It is assumed that the force F_e at the end of the attachment, representing the local plate stress σ_p in a reference area up to $t/2$ from the attachment end (t = plate thickness), is transferred through the hatched effective weld area in Fig. 3.4, section A-A. The averaged local nominal weld stress $\sigma_{n,w,loc}$ in this effective weld area due to this force F_e becomes:

$$\sigma_{n,w,loc} = \frac{F_e}{2a \cdot t + a^2} \quad (3.7)$$

Here, the weld area is conservatively assumed with throat thickness a (and not with leg length). It has been shown that the stress intensity factors of this complex 3D situation at attachment ends and of the corresponding 2D case of cruciform joints are almost the same (Fricke and Doerk, 2006). As a consequence, the fatigue assessment can be based on the same fatigue class as for cruciform joints (FAT 36 - 40 for steel).

The local nominal or structural weld stress can be determined from a finite element model with shell or solid elements in the attachment having a breadth of $t/2$ or even t . Their stresses are used to derive the force F_e from the averaged local plate stress $\sigma_{n,p,loc}$ in the reference area $t \times t/2$. As an alternative, the local nominal weld stress may be determined directly in the section A-A, where the normal stress components can be evaluated from selected weld elements of a finite element model. It should be mentioned that the load transmitted has to be referred to the effective weld area shown in Fig. 3.4. If the non-fused slit is modelled, nodal forces rather than element stresses should be evaluated in coarse finite element meshes (Fricke and Doerk, 2006). However, the evaluation of the loads transmitted is also possible when the slit is not modelled.

In the *second case*, not only normal stresses are acting perpendicular to the weld as assumed in Fig. 3.4, but also significant shear stresses parallel to the weld. These are present particularly in attachment ends with soft toes where weld root failures may also occur.

Two possibilities for their consideration have been proposed. In the first, the averaged stress components in the effective weld area are used, i. e. normal stress $\sigma_{\perp}$ and parallel shear stress $\tau_{\parallel}$, see Fig. 3.5a. They can be evaluated in the same way as described above. The following equivalent weld stress has been proposed for the fatigue assessment:

$$\sigma_{n,w,loc} = \sqrt{\sigma_{\perp}^2 + \tau_{\parallel}^2} \quad (3.8)$$

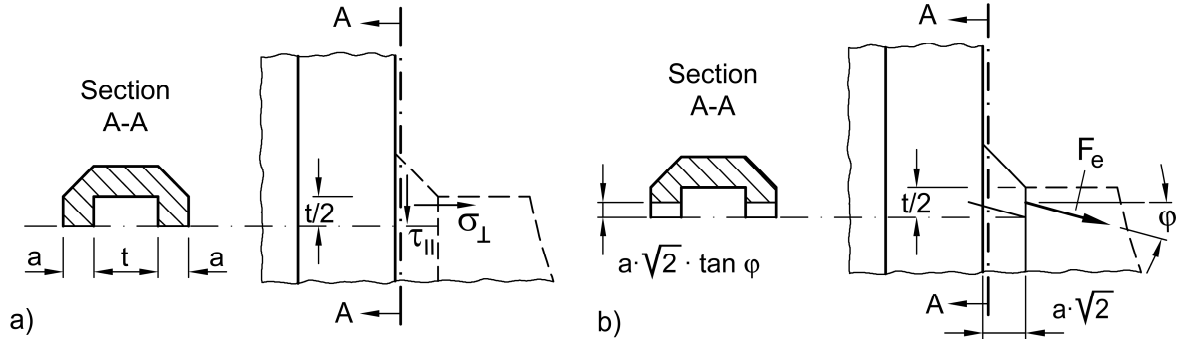


Fig. 3.5: Consideration of horizontal shear stresses using a) stress components in the weld and b) the force F_e derived from the local principal plate stress in the attachment

The alternative approach uses the resultant force F_e at the attachment end acting in the reference area $t \times t/2$ in the plate beneath the weld. This force and its angle φ from the normal may be derived from the averaged principal stresses at the nodal points in the reference area. It is assumed that the total force F_e is transferred through the effective weld area reduced according to the angle φ , see Fig. 3.5b. Then the resulting local nominal stress in the weld becomes:

$$\sigma_{n,w,loc} = \frac{F_e}{2a(t - a\sqrt{2}\tan\varphi) + a^2} \quad \text{with } \varphi \leq \arctan\left(\frac{t}{2a\sqrt{2}}\right) \quad (3.9)$$

The averaged principal stress $\sigma_{1,n,loc}$ is acting on the projection of the reference area $t \times t/2$, which is considered as follows when calculating the force F_e :

$$F_e = \sigma_{1,n,loc} \cdot (t^2/2) \cdot \cos\varphi \quad (3.10)$$

As shown in eq. (3.9), the evaluation is no more valid for large angles φ , which may happen, when the reference area for stress evaluation comes too close to the upper surface of the soft toe.

An application of the approach is demonstrated in Section 4.2.

3.3.3 Approach for local weld throat bending using stresses in the weld leg

Local weld throat bending occurs particularly in one-sided fillet or partial-penetration welds (see Fig. 2.2c and d) and is caused by eccentric load transfer or bending of the attached plate. Fricke et al. (2006) presented an approach for the fatigue assessment of fillet welds which are predominantly subjected to throat bending. They proposed to use the linearised stress in the leg plane in the extension of the root face (Section A-A in Fig. 3.6a) and not in the weld throat, because fatigue tests as well as crack propagation analyses at cruciform and hollow section joints had shown that the crack path is closer to this plane than to the weld throat section. In addition, the stress in the leg section is – in contrast to the throat section – in several cases mainly characterised by normal stresses (i. e. axial and bending stress, $\sigma_{m,w}$ and $\sigma_{b,w}$ respectively) and less by shear stresses τ_w . This facilitates the stress analysis and even allows the evaluation of linearised stresses in one-sided welds from the loads in the attached plate by equilibrium conditions as outlined below. The leg length shall be limited to the thickness of the attached plate to avoid non-conservative results.

If mainly axial and bending stresses are acting and shear stresses are small, fatigue class FAT 80 has been found from different fatigue test series to be appropriate for welds at steel. The fatigue class is higher than that mentioned in the previous sub-section, because the bending portion is now included. It should be noted, that larger shear stresses τ_w change the direction of the principal stress and, hence, the crack path. In case of doubts, the amount of τ_w and the applicability of the approach should be checked (it has been proposed that it should be less than 20% of the normal stress, which changes the principal stress direction by about 10°).

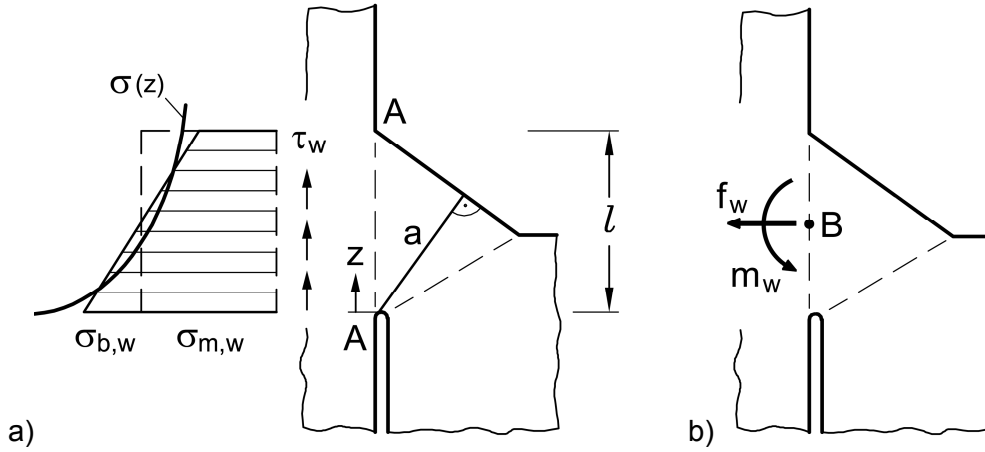


Fig. 3.6: Linearisation of stresses in the leg section of the weld (a) and internal normal force and moment per unit length in leg section (b)

Using finite elements models with solid elements

The stresses or forces in the leg section can be directly linearised to obtain the membrane and bending portion $\sigma_{m,w}$ and $\sigma_{b,w}$, respectively, of the structural weld stress $\sigma_{s,w}$:

$$\sigma_{s,w} = \sigma_{m,w} + \sigma_{b,w} \quad (3.11)$$

$$\sigma_{m,w} = \frac{1}{\ell} \int_0^{\ell} \sigma(z) dz \quad (3.12)$$

$$\sigma_{b,w} = \frac{6}{\ell^2} \int_0^{\ell} \sigma(z) \left(\frac{\ell}{2} - z \right) dz \quad (3.13)$$

where $\sigma(z)$ is the stress normal to the leg section, z the coordinate along weld leg and ℓ the leg length, see Fig. 3.6a. In the same way, also the shear stresses τ_w can be linearised.

The determination of structural weld stresses from finite element models can become unreliable due to the influence of the notch singularity at the weld root (Fricke and Kahl, 2006). Therefore it is recommended to use internal nodal forces instead of element stresses for the integration according to eq. (3.12) and (3.13), as these naturally satisfy equilibrium conditions.

If a one-sided weld is present as shown in Fig. 3.6a, the structural weld stress can generally be derived also from the membrane and bending portion σ_m and σ_b of the structural stress in the attached plate by using equilibrium conditions between the weld and the plate sections. This means that a relatively coarse mesh established for the analysis of structural hot-spot stresses can be used.

An application of this approach demonstrated in Section 4.3.

Using finite elements models with shell elements

In a *model with shell elements*, the structural stress components can be directly obtained from the internal forces and moments f_w and m_w per unit length acting at the weld leg mid-line, see Fig. 3.6b, by the following equations:

$$\sigma_{m,w} = \frac{f_w}{\ell} \quad (3.14)$$

$$\sigma_{b,w} = \frac{6 m_w}{\ell^2} \quad (3.15)$$

Obtaining f_w and m_w can also be done from an appropriate shell element model such as the one described for a one-sided weld by Turlier et al. (2010). Element nodes are located at the mid-height of the leg section (point B). The internal forces and moments are obtained by transformation of nodal forces using the shell element shape functions. Metal sheets are represented by their mid-surfaces, meshed with shell elements. In order to assess fatigue, node locations are forced at the root and the toes, see Fig. 3.7a.

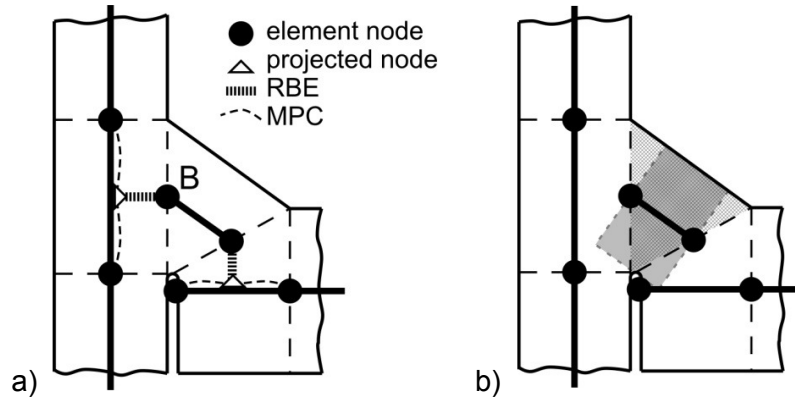


Fig. 3.7: Shell element model (a) and equivalent section (b)

The weld is represented by shell elements which are connected to the sheets using the FEA gluing technique. Weld leg nodes are projected on the mid-surfaces and linked with rigid body elements (RBE). At last, the projected nodes are coupled with sheet element nodes using multipoint constraints (MPC).

With weld throat thickness assigned as shell thickness and gluing technique avoiding additional rigidity, the model is representative regarding the stiffness of the assembly, see Fig. 3.7b. Moreover, local nominal weld stress can be determined by extracting the loads on the shell element. The shear stress component is obtained if thick shell element formulation is used.

The approach is applied to different examples in Section 4.

3.3.4 Approaches for local weld throat bending using stresses in the weld throat

A specific procedure for the fatigue assessment of weld root failure using stresses in the weld throat has been proposed by Sørensen et al. (2005), allowing various load combinations to be considered. The weld is modelled by isoparametric 20-node hexahedral elements such that two elements describe the throat section line, see Fig. 3.8. Linear elements would require a finer mesh.

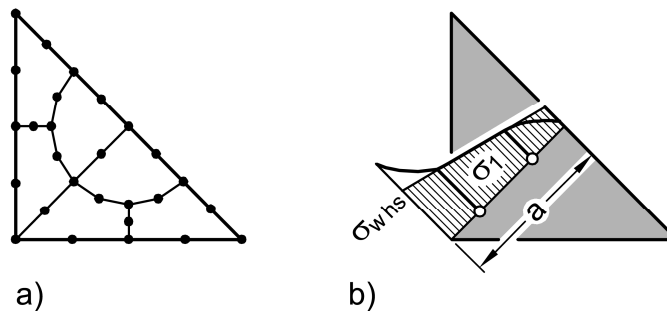


Fig. 3.8: Fillet weld modelling by 20-node hexahedral elements and evaluation of the hot-spot weld stress by linear extrapolation (after Sørensen et al., 2005)

The structural stress evaluation is based on an extrapolation of the stresses evaluated at two points located one quarter of the throat thickness apart from the weld root or weld surface, respectively. The stresses at these 'quarter points' are linearly extrapolated to the weld root, where a stress singularity exists, thus defining the 'hot-spot weld stress' $\sigma_{hs,w}$. The authors showed that in several cases it is sufficient to evaluate and extrapolate the first principal stress σ_1 at the mid-side nodes of the two elements. In more complex load cases it is recommended to extrapolate the six stress components σ_x , σ_y , σ_z , τ_{xy} , τ_{yz} , and τ_{zx} to the weld root and then to evaluate the first principal stress there. Specific meshing rules have to be observed in the neighbourhood of the weld.

The hot-spot weld stress was successfully used to describe a uniform narrow scatter band of fatigue test results for fillet-welded cover plates and cruciform specimens made of structural steel, comprising different degrees of weld throat bending. An evaluation with the prescribed inverse slope exponent $m = 3$ results in a characteristic fatigue strength of 61 N/mm^2 , however based on a probability of survival $P_s = 95\%$.

A similar approach was proposed by Hong (2010) using the axial and bending stress in the weld throat in connection with the same stress parameter used in previous proposals for the weld toe, thus considering effects of thickness and stress gradient. A similar approach has been applied to the ends of longitudinal attachments by Maddox et al. (2010). [In order to fit Maddox test results with fatigue strength of FAT 80, the idea is to transpose linearised structural stress calculation as described in 3.3.2 to any section through the weld throat.](#) Turlier et al. (2011) have developed an analytical formula based on the weld leg line forces and moments and weld throat sizes. It gives the structural stress at the weld root depending on the root weld throat section angle. The maximum structural stress may be evaluated. This has been also used in the case of asymmetric fillet weld subject to pure bending. In special cases, the stress in the plate adjacent to the weld may have additionally to be taken into account, particularly in case of high penetration joints (Turlier et al., 2011).

3.3.5 Approach using the 1-millimeter stress

Xiao and Yamada (2004) proposed to apply their structural stress approach using the 1-millimeter stress not only to weld toes, but also to weld roots. The idea behind the approach is that the stress in a distance of approx. 1 mm apart from the local notch in the direction of the expected crack propagation is representative for the early crack propagation phase which consumes the major part of the fatigue life.

The approach has successfully been applied to various fillet-welded joints with weld toe failure, resulting in characteristic fatigue strength of FAT 100. The application to cruciform joints with weld root failure indicated a FAT class reduced to approx. FAT 71.

The stress analysis requires a relatively fine finite element mesh with elements having a size of max. 1 mm (better 0.5 mm), allowing the stress evaluation at the nodal point in a distance of 1 mm from the weld root. It is not fully clear which stress component has to be taken for the fatigue assessment. In case of doubt, the first principal stress should conservatively be evaluated.

3.4 Effective Notch Stress Approach

3.4.1 Overview of the approach

The *effective notch stress approach* considers the increase in local stress at the notch formed by the weld toe or the weld root, based on theory of elasticity, i. e. without consideration of elastic-plastic material behaviour. The micro-structural support effect of the material, which considers the effect of the inhomogeneous material structure under a stress gradient on fatigue behaviour, may be taken into account by averaging the stress over a definite area.

This approach is mainly used in the form of fictitious notch rounding, see Fig. 3.9 (Radaj et al., 2006; Fricke, 2008; Hobbacher, 2009). The basic idea behind this approach is that the stress re-

duction in a notch due to averaging the stress over a certain depth can alternatively be achieved by a fictitious enlargement of the notch radius ρ .

In a 'worst case' or conservative way, Radaj assumed a fictitious radius of $r_{\text{ref}} = 1 \text{ mm}$, meanwhile termed *reference radius*. This reference radius derived for welded joints in steel is also used for welded joints in aluminium and magnesium alloys.

The notch rounding with a reference radius of 1 mm may cause problems in thin structures, less than 5 mm thick. In particular, the groove created by this radius at the weld root causes a substantial reduction in cross-section and hence modifies the stress distribution, which affects the results of the fatigue assessment.

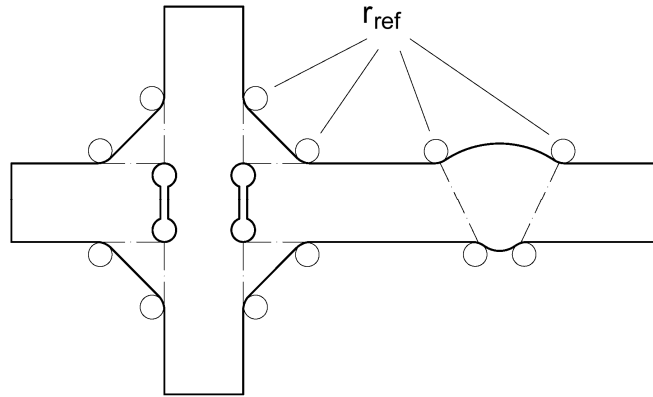


Fig. 3.9: Fictitious notch rounding (graph according to Hobbacher, 2009)

Therefore, a *small-size notch approach* which uses a reference radius $r_{\text{ref}} = 0.05 \text{ mm}$ has been proposed for assessing particularly root failure of welded joints in thin-sheet material (steel, aluminium and magnesium alloys). The re-evaluation of tests shows that it seems to be better suited also for the fatigue assessment of laser stake-welded T-joints (Frank et al., 2011)

3.4.2 Modelling and numerical analysis

The rounding of the roots of non-penetrating fillet welds is shown in Fig. 3.9. The length of the non-welded root faces is retained by locating the vertex point of the circle at the weld root, see Figs. 3.10 and 3.11. Different shapes are possible (Radaj et al., 2009), among others the keyhole notch and the U-shaped notch, Fig. 3.10b and c. The latter reduces the high stress concentration in the keyhole notch for loading parallel to the non-welded root faces but it should be noted that it can also lead to an underestimation of the required notch stress for assessing potential fatigue failure in the weld throat. Therefore, it cannot generally be recommended.

In thin-walled structures it might be necessary to compensate for the increase in net section stress due to the notch depth (Radaj et al., 2006).

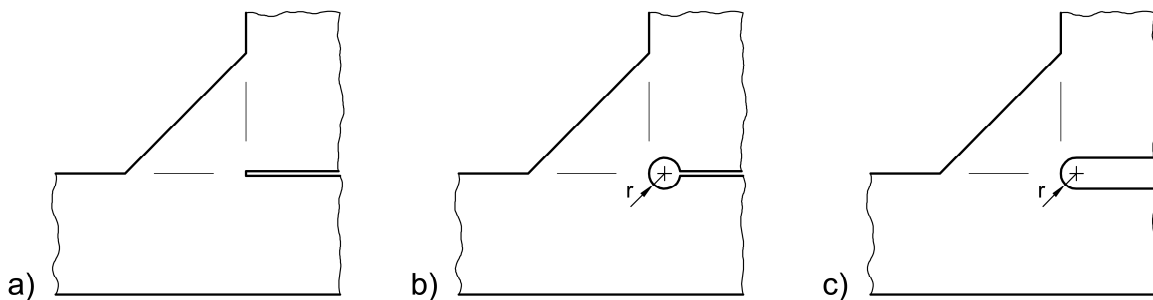


Fig. 3.10: Rounding of the weld root of a non-penetrating fillet weld by a keyhole and an U-shaped notch

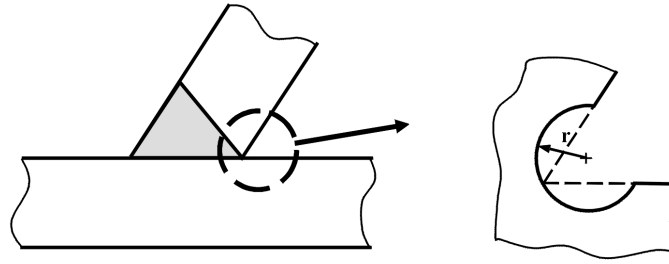


Fig. 3.11: Notch rounding of the weld root of a Y-joint

Usually, the notch stress is computed numerically using the finite element method. The stresses may be solved by 3D or a 2D analysis, the latter being restricted to special cases, where variations of the geometry and loading in the 3rd direction can be neglected. In this case, plane strain conditions are usually assumed as biaxial stresses occur in the notches due to restraint in the 3rd direction.

The discretization of the structure is normally performed such that a relatively coarse overall mesh is established, which is locally refined in the neighbourhood of the notches under consideration. The mesh should be gradually refined towards the notched area, avoiding large steps in element size and excessive element distortion. Fig. 3.12 shows a typical example for a weld root modelled with a keyhole notch.

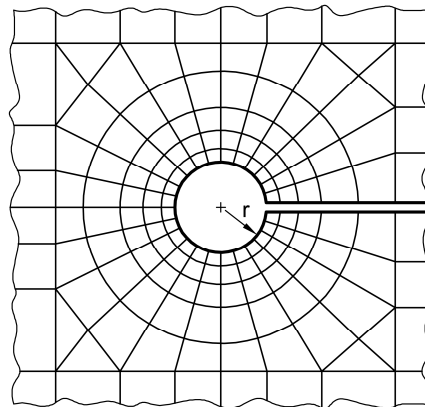


Fig. 3.12: Typical mesh for a notch stress analysis with elements having a quadratic shape function at a weld root modelled with a keyhole shape

Recommended element sizes at the notch root rounded with radius r are $r/4$ or less for elements with higher-order displacement functions and accordingly reduced for elements with linear displacement function (Fricke, 2008). It should be observed that this element size applies to the directions along the notch surface as well as perpendicular to this, in the latter preferably even with smaller size because of the steep stress gradient.

Notch stresses are evaluated on the rounded surface of the notch, using the tangential and normal stresses in the section and the shear stresses on the notch surface. From these, principal or equivalent stresses can be derived.

Further aspects for modelling and stress evaluation are discussed by Fricke (2008).

3.4.3 S-N curves and FAT-classes

The FAT values to be used for the effective notch stress approach S-N curve were derived from a statistical analysis of relevant fatigue test results obtained from welded joints and associated notch stress analyses. The results of the analyses are summarized in the left part of Table 3.1 for the reference radius $r_{\text{ref}} = 1 \text{ mm}$ and in the right part for the reference radius $r_{\text{ref}} = 0.05 \text{ mm}$. It should

be noted that these fatigue classes have been derived under the assumption that the S-N curve has a slope exponent of $m = 3$. Different slope exponents, e. g. $m = 5$, have been observed particularly in thin-walled structures, leading to suggestions for modified approaches (Sonsino et al., 2010).

The FAT classes are based on the range of maximum principal stress in the notch root. A reduction by one category is recommended in the case that the equivalent von Mises stress is assessed. This is recommended particularly for cases with pre-dominant shear loading parallel to the weld line, being characterized by principal stresses on the notch surface having different signs. Further details have been given e. g. by Fricke (2008) and Sonsino (2009b).

Table 3.1: Characteristic fatigue strength ($P_s = 97.7\%$, $N = 2 \cdot 10^6$) for welds of different materials based on maximum principal stress

Material	Characteristic fatigue strength for $r_{ref} = 1 \text{ mm}$	Characteristic fatigue strength for $r_{ref} = 0.05 \text{ mm}$
Steel	FAT 225	FAT 630
Aluminium alloys	FAT 71	FAT 180
Magnesium	FAT 28	FAT 71

3.4.4 Multiaxial fatigue

Multiaxial stress states with proportional loading can be assessed using either the range of the maximum principal stress – as long as both principal stresses have same signs - or the equivalent von Mises stress as described in the previous section. The interaction equation presented in section 3.2.4 can be used too, using the local stress components and respecting the mentioned limitations between proportional and non-proportional loading.

3.5 Approaches based on the Stress Intensity Factor

3.5.1 Overview of the approaches

The theoretically infinite stress at sharp V-notches with notch opening angle 2α can be described by a notch stress intensity factor (N-SIF) in a similar way as for crack tips. Local stress distributions in plane configurations are given as a combination of a symmetric and skew-symmetric stress field. The N-SIFs K_1 and K_2 for mode I and II loading quantify the magnitude of the asymptotic stress distribution according to the theoretical solution of Williams (1952).

The N-SIFs can directly be used to describe the crack initiation at sharp corners (Boukharouba et al., 1995; Verreman and Nie, 1996). Lazzarin and Tovo (1998) developed an N-SIF approach for welded joints and showed that fatigue test results of various welded joints form an S-N curve with reasonable scatter if based on N-SIF.

The N-SIF approach requires knowledge of the elastic stress field in the region very close to the notch tip. Similar as for the stress analysis at crack tips, finite element meshes must be very fine in this region.

However, the N-SIFs K_1 and K_2 can be related to the total elastic *strain energy density* (SED) $\bar{W}$ averaged over a cylindrical sector with a given characteristic radius R_0 in the case of the mode I and mode II stress distributions under plane strain conditions by the following equation:

$$\bar{W} = \frac{e_1}{E} \left[\frac{K_1}{R_0^{1-\lambda_1}} \right]^2 + \frac{e_2}{E} \left[\frac{K_2}{R_0^{1-\lambda_2}} \right]^2 \quad (3.16)$$

where: E = Young's modulus
 λ_1, λ_2 = the eigenvalues of the stress field for K_1 and K_2 modes (see Fig. 3.13)
 e_1, e_2 = parameters depending on the notch opening angle 2α .

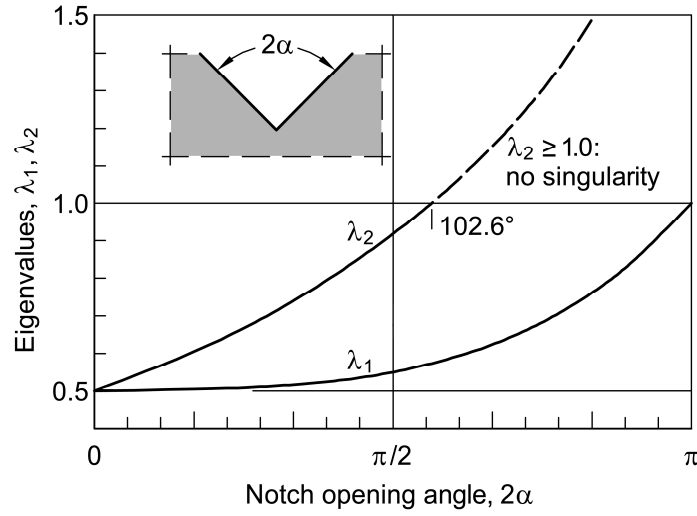


Fig. 3.13: Eigenvalues defining the order of stress singularity at sharp V-notches depending of 2α (from Radaj et al., 2006)

When 2α is 135 deg, a typical value for fillet-welded joints, the mode I stress distribution is singular ($1 - \lambda_1 = 0.326$), whereas the mode II distribution is not ($1 - \lambda_2 = -0.302$). For $2\alpha = 0^\circ$, all distributions are singular and reflect the characteristic stress field at a crack tip. The parameters e_1 and e_2 have been described analytically by Lazzarin and Zambardi (2001). For $2\alpha = 135$ deg these are $e_1 = 0.118$ and $e_2 = 0.111$ and for $2\alpha = 0$ deg $e_1 = 0.133$ and $e_2 = 0.34$.

The control radius R_0 is material-dependent and has been proposed by Lazzarin and Zambardi (2001) and Livieri and Lazzarin (2005) to be 0.28 mm considering the failure behaviour of conventional fillet-welded joints under mode I loading at steel. For an opening angle of 0 deg, this value is conservative. Corresponding considerations would result in $R_0 = 0.36$ mm.

The correlation between the average elastic strain energy density (SED) and the N-SIF offers a practical way of computing the relevant fatigue parameter with a coarser finite element model. The average SED can directly be used as fatigue parameter (*SED approach*), which allows including the results of different notches (i. e. weld toes and roots) within the same diagram (whereas the N-SIFs have different units). A presentation of the results for 650 welded joints has been given by Lazzarin et al. (2008), see Fig. 3.14 (mostly transverse non-load carrying fillet welded joints)

A uniform control radius $R_0 = 0.28$ mm has been used for both weld toe and weld root failure. The thickness of the main plate ranged from 6 to 100 mm, so that its effect is included. The scatter band given in Fig. 3.14 refers to $\pm$ two standard deviations, giving a characteristic fatigue strength of $\Delta \bar{W} = 0.058$ Nmm/mm³.

A relationship with stresses can be found by the simple expression

$$\Delta \sigma_{eq} = \sqrt{2E' \cdot \Delta \bar{W}} \quad (3.17)$$

with $E' = E / (1 - \nu^2)$ for plane strain conditions (Lazzarin et al., 2008). Eq. (3.17) describes the equivalent stress for a calculated SED averaged over a definite volume. By this the vertical axis in Fig. 3.14 could be converted into an equivalent stress range, resulting in a slope exponent $m = 3$ due to the square root in eq. (3.17).

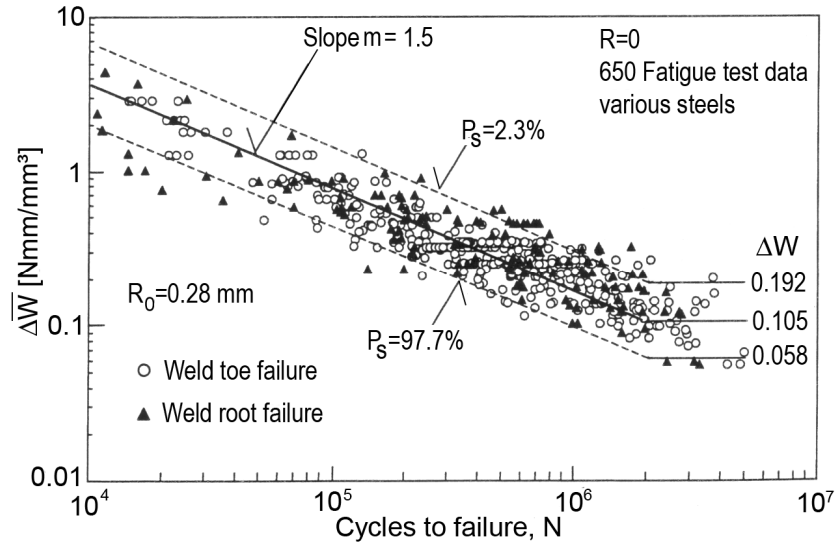


Fig. 3.14: Fatigue strength of fillet-welded joints as function of the range of the average strain energy density $\Delta \bar{W}$ (Lazzarin et al., 2008)

An alternative approach based on the N-SIF is the so-called *Peak Stress Approach* proposed by Meneghetti (2006, 2008). This approach utilizes the ratio K_{FE}^* between the notch stress intensity factor K_1 and the nodal stress σ_{peak} in the V-shaped notch for a finite element mesh with given global element size d and displacement functions, which is valid as long as the mode II stress contribution to the local stress field is absent or negligible. For a mesh where only two elements share the nodal point in the weld toe and four elements in the weld root (see Fig. 3.15), the following relationship has been found for certain linear 2D elements (PLANE42 of ANSYS element library):

$$K_{FE}^* = \frac{K_1}{\sigma_{peak} \cdot d^{1-\lambda_1}} \cong 1.38 \quad (3.18)$$

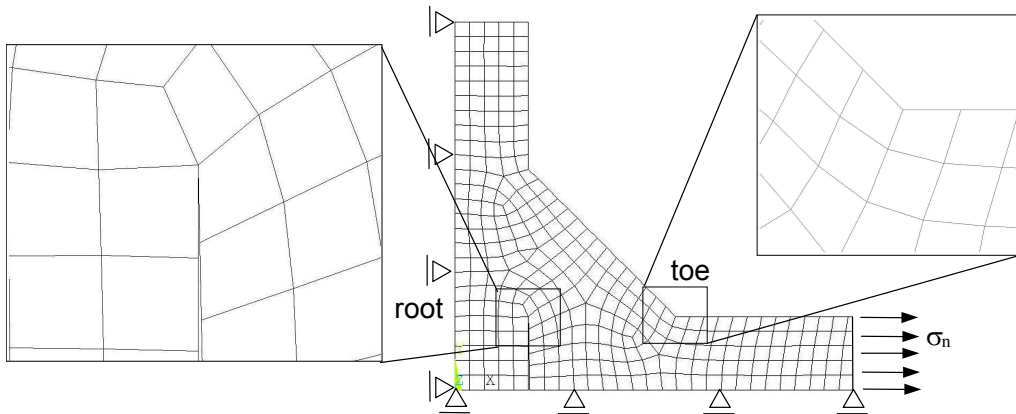


Fig. 3.15: Pattern of finite elements to analyse a welded joint by means of the Peak Stress Method (Meneghetti and Lazzarin, 2011)

It should be kept in mind that this and the following relations are only valid for the element type mentioned. The opening angle α may be between 0 and 135 deg. It has to be kept in mind that different element types and mesh patterns lead to different peak stress values which is mainly due to varying internal element stresses and stress extrapolation rules from the integration points to the nodal points.

A link between the range of the peak stress $\Delta\sigma_{peak}$ and the strain energy density SED averaged within the control radius R_0 was found by Meneghetti and Lazzarin (2011) by using the range of the equivalent stress $\Delta\sigma_{eq}$, resulting in:

$$\Delta\sigma_{eq} = f_w \cdot \Delta\sigma_{peak} = 1.38 \cdot \sqrt{\frac{2e_1}{1-\nu^2}} \cdot \Delta\sigma_{peak} \cdot \left(\frac{d}{R_0}\right)^{1-\lambda_1} \quad (3.19)$$

The range of the average strain energy density is given by eq. (3.16). The factor f_w is listed in Table 3.2 for the factor in eq. (3.19), $\nu = 0.3$, $d = 1$ mm, $R_0 = 0.28$ mm and λ_1 and e_1 acc. to Lazzarin and Zambardi (2001).

The weighted peak stress range $f_w \cdot \Delta\sigma_{peak}$ can be used as alternative parameter for an S-N curve (Meneghetti and Lazzarin, 2011).

Table 3.2: Parameters for eq. (3.19) for $\nu = 0.3$, $d = 1$ mm and $R_0 = 0.28$ mm (Meneghetti and Lazzarin, 2011)

2α (deg)	λ_1	$e_1^{(a)}$	f_w
0	0.500	0.133	1.410
90	0.544	0.145	1.392
110	0.586	0.136	1.278
120	0.616	0.129	1.198
125	0.633	0.126	1.159
135	0.674	0.118	1.064
145	0.723	0.109	0.961
150	0.752	0.104	0.905

3.5.2 Computation of notch stress intensity factors or strain energy density

Different possibilities exist to compute the stress parameter for the application of the notch stress intensity factor approach described above:

- Direct computation of the notch stress intensity factor N-SIF. As mentioned before, this requires an extremely fine finite element mesh (Tovo and Lazzarin, 1998) from which the stress distribution can be compared with the theoretical solution of V-notches to derive the N-SIFs.
- Computation of the average strain energy density (SED) in a control volume with radius R_0 allowing the N-SIF to be derived (eq. 3.16). Alternatively, the SED or the equivalent stress can directly be used for the fatigue assessment. The approach requires finite element sizes of R_0 or less.
- Lazzarin et al. (2008) have shown that the computation of the average SED for a larger control radius of $R^* = 1$ mm and converting it to 0.28 mm using eq. (3.16) yields results with sufficient accuracy at the weld toe where only mode I stress distribution is singular. In principle, eq. (3.16) can be used also under mixed mode conditions, when the degree of singularity of the mode I and mode II stress fields is the same, like it occurs at the weld root region ($\lambda_1 = \lambda_2 = 0.5$).

- However, at the roots of non-penetrating welds with loading parallel to slit the effect of the T-stress (i. e. the constant stress component parallel to the slit (Williams, 1957) has to be taken into account. This means that the main contribution of the T-stress σ_T to the strain energy density at weld roots, i. e.

$$\Delta \bar{W}_T = \sigma_T^2 / 2E' , \quad (3.20)$$

has to be deducted from the average strain energy $\Delta \bar{W} (R^*)$ before converting the strain energy density by eq. (3.16). Doing so, it is possible to compute the increase to the local strain energy density due only to the singular stress fields. After conversion ΔW_T has to be included again. The calculation remains approximate because the effect of further terms (Lazzarin, 2009) has not been considered here for the sake of simplicity.

- Computation of the peak stress σ_{peak} and converting it into the notch stress intensity factor (eq. 3.18) or the SED (eq. 3.19 and 3.17). A typical FE mesh has been shown in Fig. 3.15.

Fig. 3.16 shows a fine-meshed FE model for the 2D analysis of the average SED in a control volume with $R_0 = 0.28$ mm at the weld toe as well as weld root. The alternative with a coarse mesh is illustrated in Fig. 3.17, using triangular elements around the notch tip having quadratic shape function and a length of 1 mm. Suggestions for meshing and several applications have been shown by Lazzarin et al. (2008) and Fischer et al. (2010).

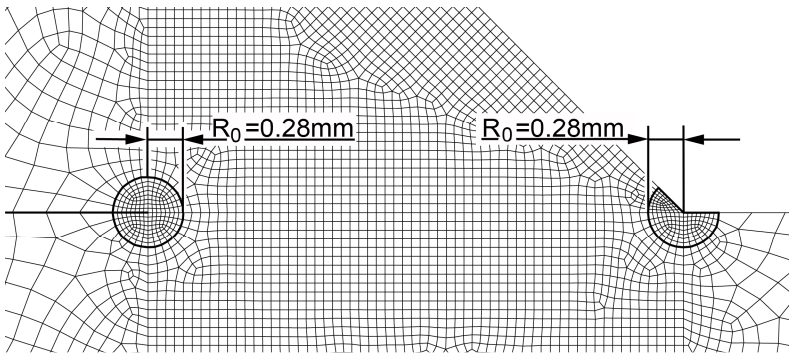


Fig. 3.16: FE model for the computation of the SED at a weld toe (right) and weld root (left)

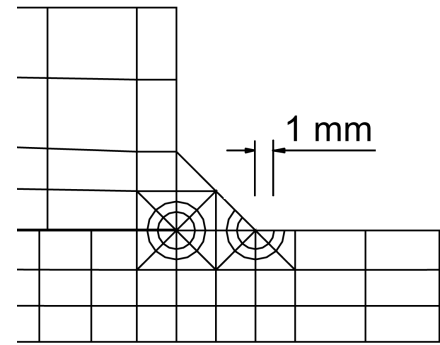


Fig. 3.17: Coarse mesh for the SED computation

3.6 Crack Propagation Approach

3.6.1 Overview of the approach

The crack propagation approach allows the phase of crack growth between an initial and a final crack length to be evaluated. Basis is the *linear elastic fracture mechanics* (LEFM) as plastic zones are rather small during fatigue crack propagation. The crack propagation rate da/dN can be estimated using the stress intensity factor at the crack tip and the power law by *Paris and Erdogan*, describing the linear relationship between the crack propagation rate and the range of the stress intensity factor ΔK in bi-logarithmic scale:

$$\frac{da}{dN} = C \cdot \Delta K^m \quad (3.21)$$

with the material parameters C and m . The linear relationship is valid between the threshold value ΔK_{th} with no crack propagation beneath and the transition to unstable fracture with the critical stress intensity factor K_c , see Fig. 3.18. Different variants exist to consider e. g. the stress ratio R .

The approach well-suited for the fatigue assessment of welded joints where the crack initiation life is short and the fatigue life is mainly determined by the crack propagation phase. This is particularly the case for weld root fatigue where the slit between the non-fused root faces acts like an initial crack.

In the following, some remarks are given on the application of the approach to weld root fatigue. Further information on the approach can be found in the relevant literature (e. g. Carpinteri, 1994; Anderson, 1995, Schijve, 2001, Radaj et al., 2006 and Hobbacher, 2009).

3.6.2 Stress intensity factors

In a first step, crack intensity factors have to be determined for different crack lengths or shapes. Different possibilities exist, such as the use of tabulated factors in handbooks and the application of numerical methods. Here, the stress intensity factors can be determined indirectly by the so-called weight factor method or directly by the finite element method using the strain energy release rate during virtual or real crack extension or the J-integral.

Finite element models have to be very fine-meshed around the crack tip. If isoparametric elements with mid-side nodes are used, a special feature can be used to improve the results, i. e. shifting the mid-side nodes towards the crack tip to the quarter points in all elements around the crack tip (preferably elements of triangular shape), see Fig. 3.19. In this case they fulfil the $r^{0.5}$ -singularity of the local stress field.

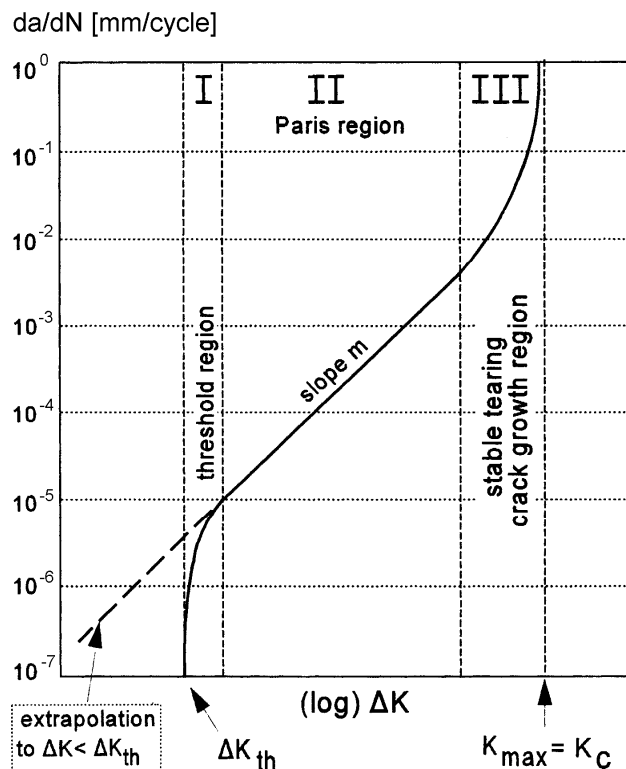


Fig. 3.18: Relationship between the crack growth rate da/dN and the range of the stress intensity factor ΔK (Radaj et al., 2006)

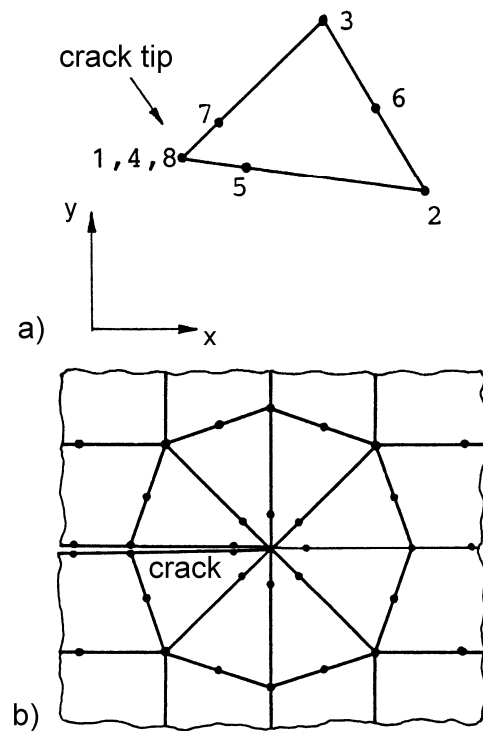


Fig. 3.19: Isoparametric element a) with shifted midside nodes (quarter point) and b) modelling

Important is the consideration of the local geometry around the crack and the loading which is taken into account by the so-called *geometry factor* in standard cases. Also the crack shape and crack path may affect the results considerably. The crack may be assumed as a continuous 2D crack or a local crack with semi-elliptical shape. If a long non-fused slit acts as an initial crack, a continuous 2D crack is usually assumed. The crack path is mainly determined by the ratio between

the stress intensity factors for the sliding mode K_{II} and opening mode K_I . A pronounced K_{II} results in a deviation of the crack path into a direction which is perpendicular to the max. principal stress (range).

3.6.3 Determination of the fatigue life

The fatigue life consumed by crack propagation is determined by integrating the power law (3.21) of Paris and Erdogan. Material constants C and m as well as ΔK_{th} are proposed in different guidelines (e. g. Hobbacher, 2009; BS 7608, 1993), assuming upper bound values for predictions on the safe side.

The critical crack length or stress intensity factor K_c affects the crack propagation only marginally as the crack growth rate is already very high when the linear relationship (3.21) approaches K_c .

More important may be crack closure making the stress cycle only effective during the portion where the crack is open. Due to local plastification, this can be smaller than the tensile part of the stress cycle. Usually, an effective part of the range of the stress intensity factor ΔK_{eff} is defined for the determination of the crack growth rate according to eq. (3.21).

4 Demonstration Examples

In total, six demonstration examples are described. Table 4.1 gives an overview about the different approaches applied. For each approach at least two examples are given.

Table 4.1: Demonstration examples and approaches applied

Example	Nominal stress approach	Structural stress approach	Eff. notch stress approach	Approach based on N-SIF	Crack propagation approach
Cruciform joint	X	X	X	X	X
Fillet weld around attachment end		X		X	
One-sided fillet weld around RHS joint	X	X	X		
Lap joints and cover plates	X		X	X	X
Fillet-welded pipe penetration	X		X		
Laser-stake welded T-joint	X		X	X	X

4.1 Cruciform Joint

4.1.1 Description of the detail

The cruciform joint with partial penetration shown in Fig. 4.1 (and Fig. 2.3) is selected as example. The chosen geometry parameters are:

Plate thickness $2b = 25.0 \text{ mm}$
 Length of root face $2a = 12.5 \text{ mm}$
 Leg length $c = 17.7 \text{ mm} (= 12.5 \cdot \sqrt{2} \text{ mm})$

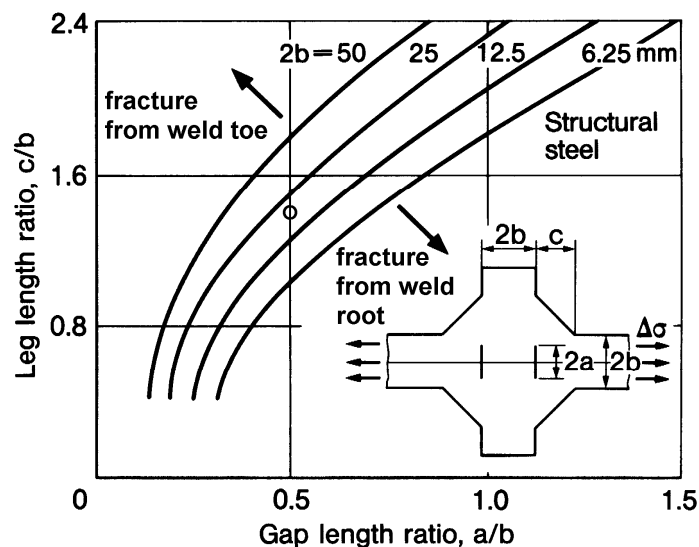


Fig. 4.1: Limit curves separating weld toe and root fracture in cruciform joints (Maddox, 1974), the geometry investigated here is indicated by a circle

In Fig. 4.1, this geometry is shortly below the limit curve for 25 mm so that the fatigue strength computed for the weld toe should be slightly above that for the weld root. The characteristic fatigue strength based on nominal axial stress range shall be determined for weld toe and weld root failure.

4.1.2 Application of the nominal stress approach

In the nominal stress approach, weld toe failure in partial penetration cruciform joints is frequently associated with the fatigue class FAT 63, see No. 413 in the IIW Fatigue Design Recommendations (Hobbacher, 2009). Weld root failure is, however, associated with FAT 36, based on the nominal weld stress. If the throat thickness is defined by the shortest distance between the end of the slit and the weld surface, i.e. 16.9 mm in the example, the ratio between the stresses in the weld throat and plate is 0.739, so that the fatigue class for weld root fatigue becomes FAT 49 if based on the plate stress. This means that weld root fatigue is more critical than weld toe fatigue which agrees well with Fig. 4.1.

4.1.3 Application of the structural stress approach

The 2D finite element model shown in Fig. 4.2 was created assuming plane strain condition. The axial load is applied at the right end ($\sigma_n = 100$ MPa).

For fatigue assessment of the *weld toe*, stress values were extracted at a distance $0.4t$ and $1.0t$ on the plate surface and linearly extrapolated to the weld toe (Hobbacher, 2009), yielding a structural hot-spot stress of $\sigma_{hs} = 100.8$ MPa.

For fatigue assessment of the *weld root*, nodal forces were extracted in the vertical weld leg section and summed up according to Fig. 3.6 and eq. (3.12 - 3.13). This gives the following results:

- membrane stress in weld leg $\sigma_{m,w} = 52$ N/mm²
- bending stress in weld leg $\sigma_{b,w} = 67$ N/mm²
- structural stress in weld leg $\sigma_{s,w} = 128$ N/mm²

The transverse shear stress in the weld is within the limits of the approach ($\tau_w = 14$ N/mm²), i. e. less than 20% of the structural stress.

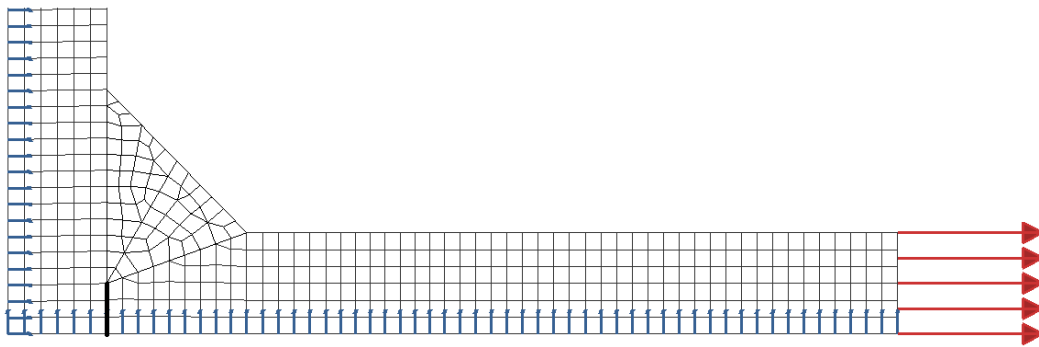


Fig. 4.2: Finite element model (quarter model), boundary conditions and loading

The structural stress at the weld toe, $\sigma_{hs} = 100.8$ MPa, and at the weld root $\sigma_{s,w} = 128$ MPa are compared with their corresponding fatigue class (FAT90 at toe and FAT80 at root). The ratios between structural stress and corresponding fatigue class show an advantage for root failure. Based on nominal stress in the main plate, the characteristic fatigue strengths are 89 MPa at the weld toe and 62.5 MPa at the weld root.

The alternative with shell elements described in Section 3.3.3 shows smaller bending stresses in the weld, hence it is on the non-conservative side-

4.1.4 Application of the effective notch stress approach

The 2D finite element model shown in Fig. 4.3 has been created, representing one quarter of the joint. Symmetry conditions have been utilized at the left and lower boundary. 2D plane strain elements with quadratic shape function have been used. The mesh fineness in the critical points, i.e. in the weld toe and weld root (modelled by a keyhole notch) corresponds to the recommendations by Fricke (2008). An axial load is acting on the right side.

The following fatigue notch factors K_f at the weld toe and root based on the stress in the abutting plate were computed:

- Weld toe: $K_f = 3.75$
- Weld root: $K_f = 4.43$

The notch stress at the weld root is about 18% higher than that at the weld toe which agrees with Fig. 4.1 where weld root fatigue is more critical.

4.1.5 Application of the strain energy density (SED) approach

The calculation of the average strain energy density (SED) $\bar{W}$ averaged within a control radius of $R_0 = 0.28$ mm with the finite element model shown in Fig. 4.4 yields the following values at the weld toe and root for an axial stress of 100 N/mm^2 :

- Weld toe: $\bar{W} = 0.192 \text{ Nmm/mm}^3$
- Weld root: $\bar{W} = 0.224 \text{ Nmm/mm}^3$

Here, the SED at the weld root is about 17% higher than that at the weld toe.

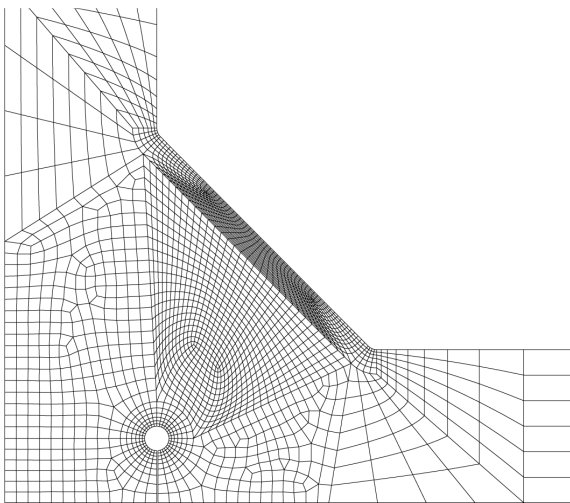


Fig. 4.3: Finite element model (1/4 model) for numerical analysis of the effective notch stress

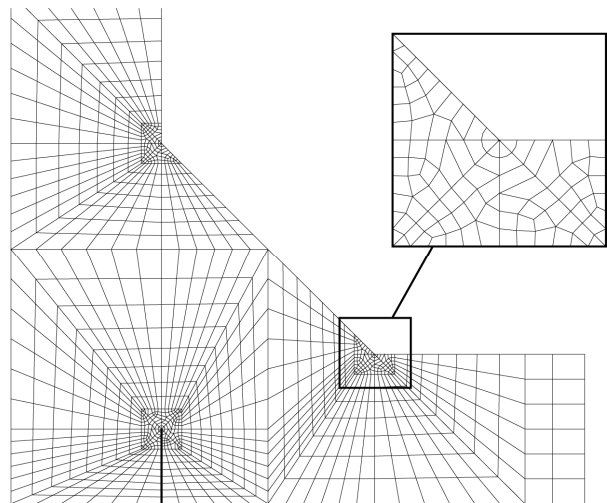


Fig. 4.4: Finite element model (1/4 model) for numerical analysis of the average strain energy density

4.1.6 Application of the Peak Stress Approach

According to Meneghetti and Lazzarin (2011), the weighted peak stress is determined at the toe and root of the joint. A free mesh of four-node quadrilateral elements (PLANE 42 elements of the ANSYS element library) shown in Fig. 4.5 was adopted in the numerical analysis and the global element size was set to 1 mm. The maximum principal stress at the toe and root locations were determined and then multiplied by the coefficient f_w reported in Table 3.2. The resulting equivalent stresses obtained for an axial stress of 100 N/mm^2 are included in Fig. 4.5, which show that root failure is predicted rather than toe failure with a stress being 18% higher.

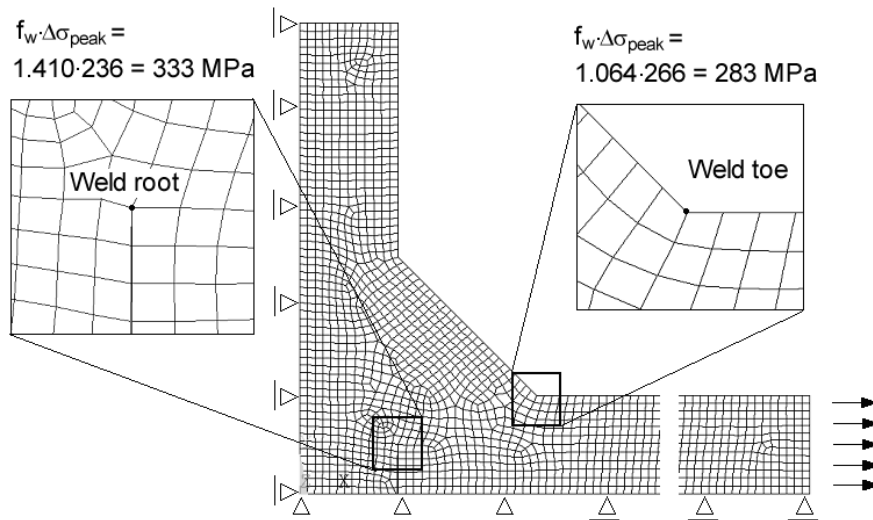


Fig. 4.5: The Peak Stress Method applied to estimate toe or root failure of a cruciform joint

4.1.7 Application of the crack propagation approach

The crack propagation approach is applied using the Paris-Erdogan law (3.21) with material parameters $m = 3$ and $C = 5.21 \cdot 10^{-13}$ (units: N and mm) recommended by Hobbacher (2009), representing the upper limit of the scatter band (2.3% probability of survival). The fatigue life is computed for a constant stress range $\Delta\sigma_n = 100 \text{ N/mm}^2$. End of life is defined when the crack length reaches half the plate thickness for toe cracks and half the throat thickness for root cracks. The remaining lifetime is rather short.

Crack propagation from the weld root

The length of the weld root face (12.5 mm) is assumed as initial crack length. A continuous 2D crack is assumed for the whole propagation phase so that a 2D model is sufficient for the numerical analysis of the stress intensity factor (SIF). The finite element method was applied using the program FRANC2D (2004). Fig. 4 shows the basic mesh created with 2D elements having quadratic displacement function. Plane strain was assumed. Symmetry conditions were utilized in two planes resulting in a quarter model.

Fig. 4.6a shows also the computed crack path, which is slightly curved due to the presence of K_{II} . The fatigue life up to a crack length of half the weld throat thickness is 265,000 cycles.

Crack propagation from the weld toe

Two numerical analyses were performed. The first assumes again a continuous 2D crack. Fig. 4.6b shows the basic mesh created for the computation in FRANC2D (2004), indicating again a curved crack front.

The initial crack length was assumed as $a_i = 0.15 \text{ mm}$ as recommended by Hobbacher (2009). The resulting fatigue life up to a crack length of half plate thickness was found to be 143,000 cycles. This is much shorter than the crack propagation life of the weld root, which does not correspond to Fig. 4.1. Even a much smaller initial crack length of $a_i = 0.05 \text{ mm}$ does not change this as the fatigue life would increase only to 158,000 cycles.

A possible explanation for the unexpected result is the assumption of a continuous 2D crack assumed at the weld toe, whereas semi-elliptical cracks initiating from small defects or high notch effects are often observed in practice. Therefore, a second analysis the SIFs with a 3D model was performed, using the program ANSYS 13.0 offering an iterative approach for the development of the crack front.

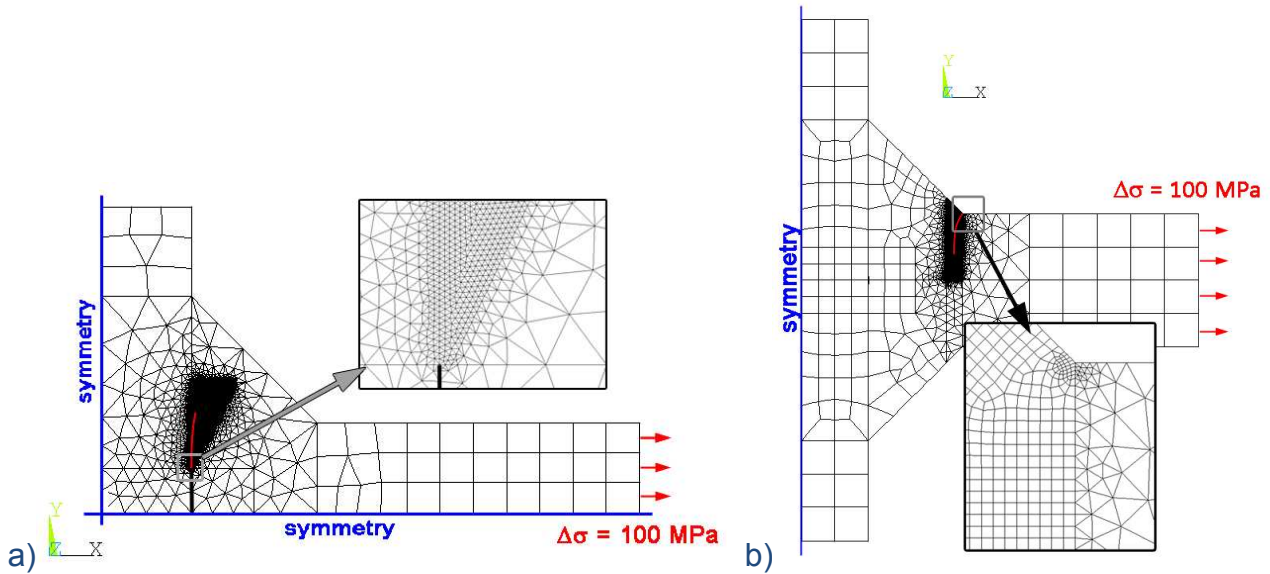


Fig. 4.6: Finite element models for numerical analysis of stress intensity factors (a) at the weld root and (b) at the weld toe

Fig. 4.7 shows the finite element model chosen. A width of the model was assumed to be 50 mm being typical for fatigue test specimens. The model consists of solid elements with quadratic displacement function. Along the semi-elliptical crack front, elements with mid-side nodes shifted towards the crack front, i.e. to the quarter-points, were used to model the stress singularity, see Fig. 3.19. Symmetry conditions were utilized at half width.

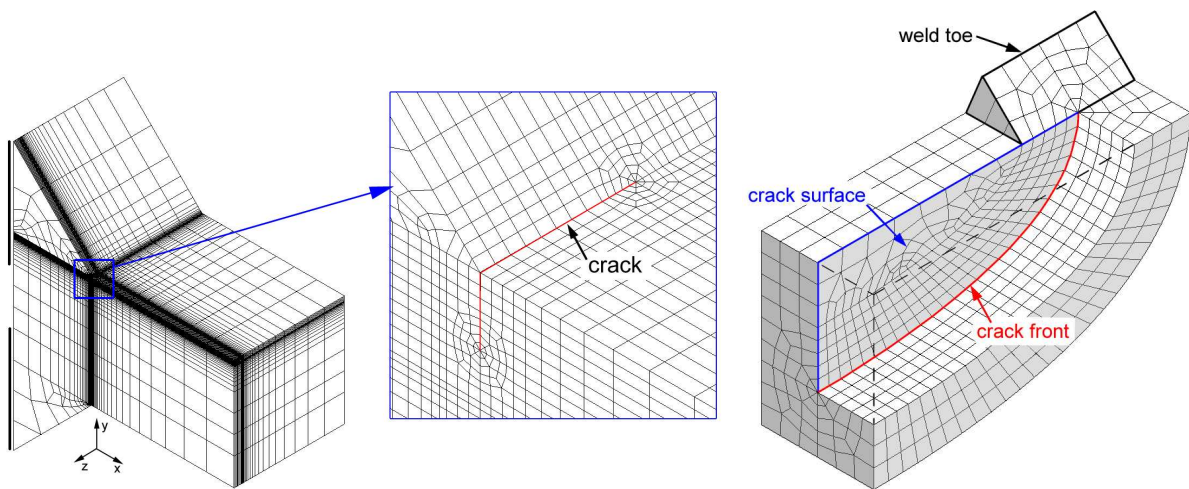


Fig. 4.7: Finite element model (half model) for numerical analysis of stress intensity factors at the weld toe (3D model) and details of crack modelling

The stress intensity factors of the semi-elliptical shape were again computed at the vertex points. In order to avoid any influence of the singularity of the sharp weld toe at the outer vertex point (see right part of Fig. 4.7), the stress intensity factor at this point has been calculated by quadratic extrapolation of the adjacent results at the crack front.

The initial crack has been assumed to be semi-circular with $a_i = c_i = 0.15 \text{ mm}$. The development of the crack front was assumed in accordance with the computed stress intensity factors at the vertex points, using the iterative approach mentioned. The iteration ended with a crack depth of 3.6 mm at 348,000 load cycles. Afterwards, a 2D crack is assumed resulting e.g. from coalescence of neighbouring cracks. This 2D crack reaches half plate thickness after a total of 380,000 load cycles.

The crack propagation life calculated in the 3D analysis with semi-elliptical crack shapes is much longer than that computed with 2D cracks. Furthermore, it is also longer than computed life for the weld root crack (265,000 cycles) which is in agreement with Fig. 4.1.

4.1.8 Comparison of the results

A comparison between all results can be performed on the basis of the characteristic fatigue strength $\Delta\sigma_c$ at $2 \cdot 10^6$ cycles for a survival probability $P_s = 97.7\%$ referring to *nominal plate stress* which can be derived from the computed local stress or SED, the assumed nominal stress and the characteristic value of the S-N curve. The latter has been assumed as FAT 225 for the effective notch stress and 0.058 Nmm/mm^3 for the characteristic SED, see Fig. 3.14. Regarding the Peak Stress Method, the SED can be derived from the equivalent stress using eq. (3.17). All approaches except for the structural stress approach show that weld root failure is more probable in agreement with Fig. 4.1. There are some differences between the results, particularly the structural stress approach yields too high fatigue strengths when compared with the test results evaluated by Maddox (1974).

Table 4.2: Characteristic fatigue strength $\Delta\sigma_c$ based on nominal stresses in the main plate

	$\Delta\sigma_c [\text{N/mm}^2]$	
	at weld toe	at weld root
Nominal stress approach	63	49
Structural stress approach	89	63
Effective notch stress approach	60	51
Strain energy approach	55	51
Crack propagation approach	57	51
Fatigue test results based on Maddox (1974)	68	63

The results of the approaches beyond the nominal stress approach do not yet consider the effects of axial misalignment. If a misalignment of 15% of the plate thickness is assumed, the secondary bending stress is particularly high at the *weld toe*, resulting in a stress magnification of $k_m = 1.45$ according to eq. (3.1). For the *weld root*, eq. (3.2) yields $k_m = 1.09$. This would mean that the results of the structural and nominal stress approaches are closer together. The notch stress, strain energy and crack propagation approaches are more conservative than the aforementioned ones and would expect crack initiation at the weld toe. Their assessment might be valid as it remains unclear to what extent axial misalignment was present in the tests being the basis of Fig. 4.1.

4.2 Fillet Weld around Attachment End

4.2.1 Description of the detail

Fig. 4.8 shows a specimen designed for the investigation of weld toe and root failure at attachment ends. The load in the plate is transferred into the attachments by shear stresses parallel to the weld having a throat thickness $a = 4 \text{ mm}$. In addition, pronounced bending stresses are acting in the attachments. The fatigue-critical detail is the soft nose of the right attachment in Fig. 4.8, where the fillet weld is subjected to axial and shear stresses.

As the detail is relatively complex, only the structural stress and the stress intensity approaches are applied, allowing the fatigue assessment of the weld toe and the weld root at the end of the attachment to be performed.

4.2.2 Fatigue tests

Constant amplitude fatigue tests were performed with a total of

- 20 specimens in as-welded state with a nominal stress range $\Delta\sigma_n = 28 \text{ N/mm}^2$ and 20 N/mm^2 at a stress ratio $R \approx 0$
- 10 specimens in as-welded state with $\Delta\sigma_n = 23.5 \text{ N/mm}^2$ at $R = 0.5$
- 10 specimens in stress-relieved with $\Delta\sigma_n = 20 \text{ N/mm}^2$ at $R \approx 0$

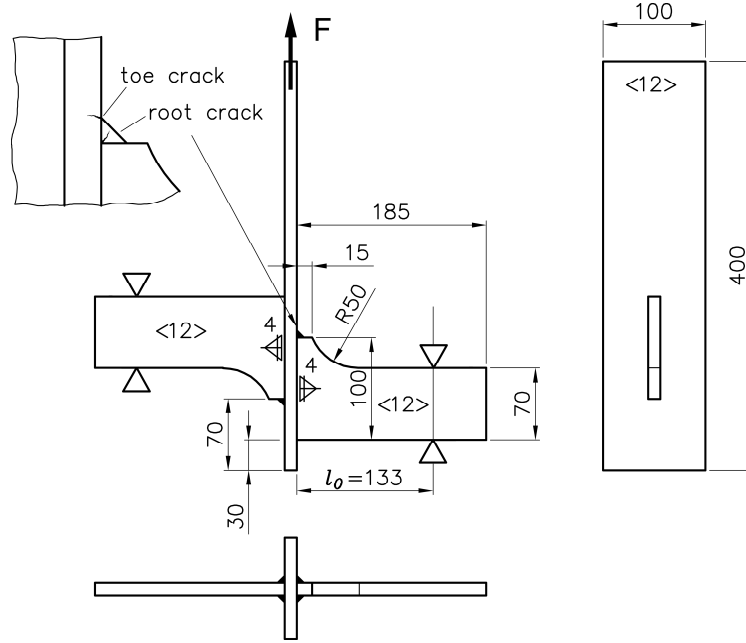


Fig. 4.6: Specimen with attachments on a plate

The tests, which are described by Fricke and Doerk (2006), showed weld root failure in almost all specimens. The detailed results will be used in the subsequent investigations for comparison with the approaches.

4.2.3 Application of the structural stress approach with solid elements

The stresses were computed for a force in the vertical plate of $F = 50 \text{ kN}$, corresponding to a nominal stress of $\sigma_n = 41.7 \text{ N/mm}^2$. A relatively coarse finite element model was created which is shown in Fig. 4.9 together with the deformations and distribution of the max. principal stress.

The structural hot-spot stress at the upper weld toe, calculated acc. to Hobbacher (2009), amounts to $\sigma_{hs} = 267 \text{ N/mm}^2$, which means a structural stress concentration factor $K_s = 6.4$ referring to the nominal stress in the vertical plate strip.

The bending moment in the transverse attachment creates a *nominal* bending stress in the weld throat of $\sigma_{n,w} = 183 \text{ N/mm}^2$, based on the section modulus of the weld throat area concentrated at the root line. More relevant is the *local nominal* stress at the end of the weld, which has been derived with the approach illustrated in Fig. 3.5b. An inclination angle of $\varphi = 32.6^\circ$ was found for the principal stress in the reference area ($t \times t/2$). The local nominal stress in the effective weld area acc. to eq. (3.9) results in $\sigma_{n,w,loc} = 346 \text{ N/mm}^2$, which is almost twice as high as the nominal stress in the weld. The evaluation of the weld stresses acc. to eq. (3.8) and Fig. 3.5a gave almost the same result (337 N/mm^2).

The fatigue assessment in Table 4.3 is performed by comparing the local stress ranges for a cyclic force of $\Delta F = 50 \text{ kN}$ and the corresponding FAT classes. FAT 90 is assumed here for partial-load

carrying weld toes on a plate surface. The comparison of the numbers shows that root cracking is much more probable than toe cracking due to the smaller ratio between endurable stress (FAT class) and applied stress range.

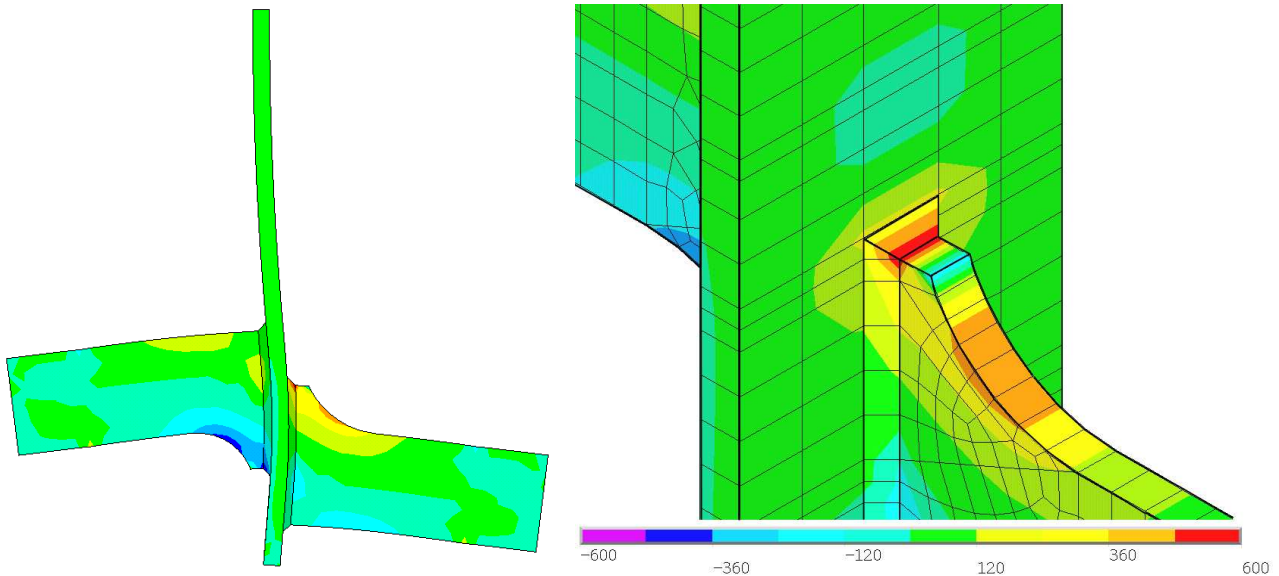


Fig. 4.9: Finite element model of the specimen with deformations and max. principal stress distribution

Table 4.3: Local stress ranges for $\Delta F = 50$ kN and characteristic fatigue strength (FAT)

Crack initiation site	Nominal stress range $\Delta\sigma_n$	Computed local stress range	FAT class (local approach)
weld toe	41.7 N/mm ²	$\Delta\sigma_{hs} = 267$ N/mm ²	90
weld root	41.7 N/mm ²	$\Delta\sigma_{n,w,loc} = 346$ N/mm ²	40

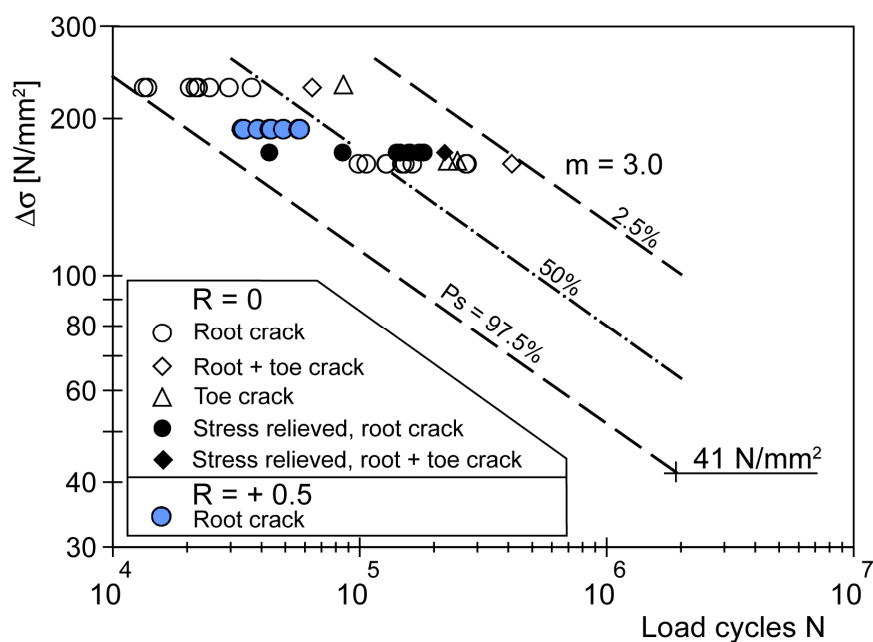


Fig. 4.10: S-N results from tests ($\Delta\sigma$ = local nominal stress in weld)

The fatigue lives from the tests are plotted in Fig. 4.10 in relation to the range of the local nominal weld stress $\Delta\sigma_{n,w,loc}$. They show that the results are only slightly affected by the stress ratio R and the residual stresses, which is in contrast to relatively stiff attachment ends where compressive residual stresses have been observed in the attachment.

The figure also shows that FAT 40 is a good basis for fatigue assessment as the characteristic fatigue strength of the test series is slightly above this class. This was also shown for other tested geometries with different toe radii and weld throat thickness (Fricke and Doerk, 2006).

4.2.4 Application of the structural stress approach with shell elements

For the same load condition ($F = 50$ kN), the finite element analysis has been performed using shell elements and the technique described in Fig. 3.7. Mesh nodes have been introduced at the $t/2$ reference locations according to Fig 3.4. The von Mises stress distribution is displayed in Fig 4.11a. Structural hot-spot stress is calculated using through-thickness forces integration over the weld boundary as shown in Fig 4.11b. The structural hot-spot stress range in the plate at the attachment end has found to be $\Delta\sigma_{hs} = 254$ N/mm² (5% less than in the solid model).

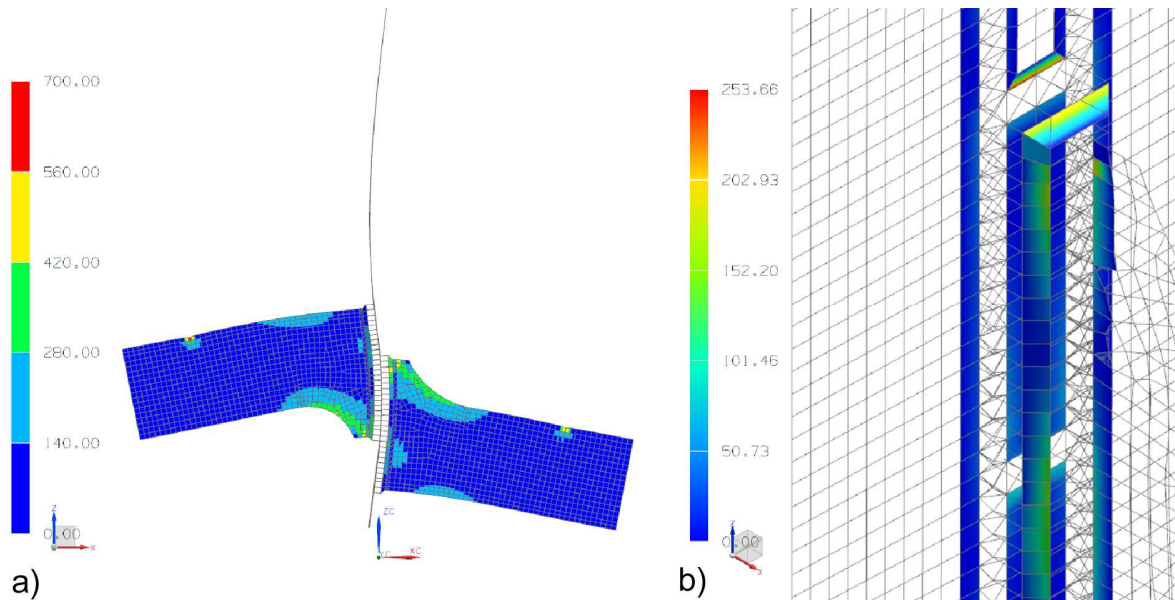


Fig. 4.11: Shell element model of the specimen with deformations and von Mises stress distribution (a) and structural stress range over the weld contour (b) in MPa

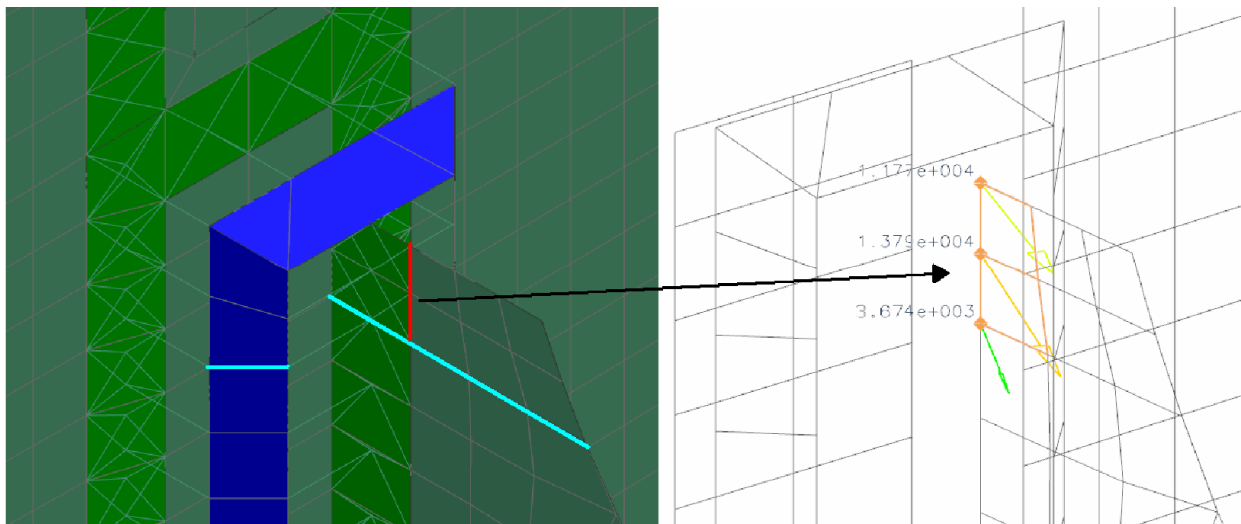


Fig. 4.12: Nodal forces extracted from the shell model

The local nominal weld stress is calculated with the forces extracted from the attachment (see Fig. 4.12) and eq. (3.9) with $\varphi = 36.4^\circ$, resulting in $\sigma_{n,w,loc} = 368 \text{ N/mm}^2$ (6% more than the solid model).

Related to their respective fatigue class FAT90 for the weld toe and FAT40 for the weld root, the highest ratio is obtained at the weld root where the crack might occur. The stresses calculated with shell elements are slightly larger than those from the solid model described above.

4.2.5 Application of the strain energy density (SED) approach

The strain energy density approach based on the notch stress intensity factor has been applied to this detail by Lazzarin et al. (2009). A nominal stress of $\sigma_n = 100 \text{ N/mm}^2$ was assumed in the vertical plate strip. The strain energy density has been evaluated by means of 3D finite element analysis using 20 node brick elements. A 3D circular sector with $R^* = 1 \text{ mm}$ surrounding the crack initiation points has been used and then reconverted to $R_0 = 0.28 \text{ mm}$ by means of the following equation based on eq. (3.15) neglecting K_2 .

$$\frac{\bar{W}(0.28\text{mm})}{\bar{W}(1.0\text{mm})} = \left(\frac{0.28}{1.0} \right)^{2(\lambda_1-1)} \quad (4.1)$$

Table 4.4 shows the computed strain energy densities for the weld toe and weld root. It turns out that the weld root is more critical (for $R_0 = 0.28 \text{ mm}$). Fig. 4.13 shows the test data in an S-N diagram based on the range of the strain energy density $\Delta\bar{W}$. The results agree well with the scatter band proposed by Lazzarin et al. (2008), see also Fig. 3.13.

Table 4.4: Mean values for the average strain energy density $\Delta\bar{W}$

Crack initiation site	Nominal stress range $\Delta\sigma_n$	$\Delta\bar{W}$ for $R^* = 1 \text{ mm}$	$\Delta\bar{W}$ for $R_0 = 0.28 \text{ mm}$
weld toe	100 N/mm ²	6.546 Nmm/mm ³	15.010 Nmm/mm ³
weld root	100 N/mm ²	5.329 Nmm/mm ³	19.030 Nmm/mm ³

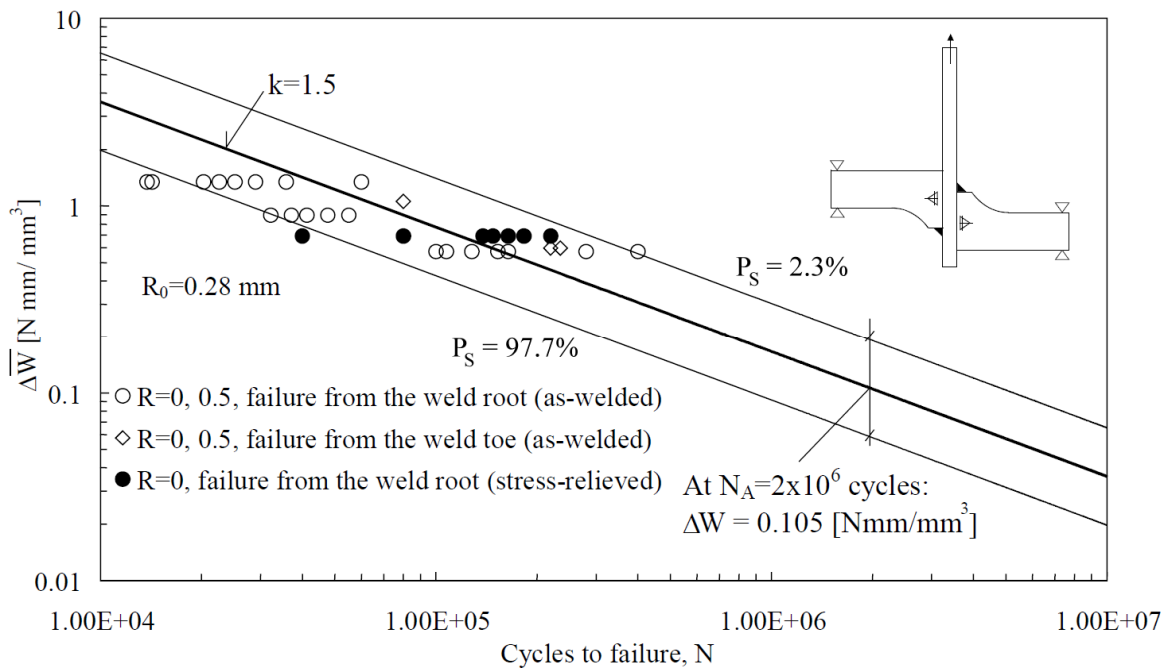


Fig. 4.13: S-N results in terms of the range of the average strain energy density $\Delta\bar{W}$ (Lazzarin et al., 2009)

4.3 One-Sided Fillet Weld around Rectangular Hollow Section Joint

4.3.1 Description of the detail

The third detail is a fillet-welded connection between two rectangular hollow sections (RHS), see Fig. 4.14. The dimensions of the RHS are 120 mm x 80 mm x 6 mm (height x width x thickness). Two hollow sections are connected to a 15mm thick intermediate plate by a single-sided fillet weld. Two types of welding process were applied: manual metal-arc welding with covered electrode, resulting in a mean throat thickness of $a = 4.3$ mm and the MAG process with flux-cored wires resulting in $a = 5.1$ mm. The gap between the intermediate plate and the hollow section is assumed to be zero, see “DET A” in Fig. 4.14.

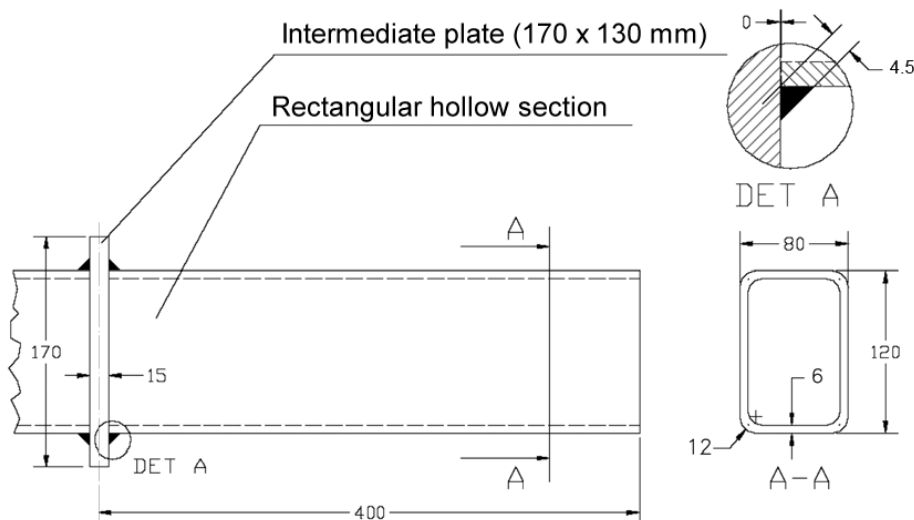


Fig. 4.14: Sketch of the fillet-welded end connection of an RHS member

Two load cases were investigated:

1. axial load at the end of the RHS, applied to 9 of the manual metal-arc welded specimens
2. vertical 3-point bending, applied to 6 of the manual metal-arc welded specimens and 7 MAG-welded specimens

In the stress analysis, symmetry conditions were utilized and a constant stress of 100 N/mm^2 was applied to the right end. In the bending load case a distributed shear force of 17.25 N was applied to the right end creating the nominal bending stress of 100 N/mm^2 at the weld.

4.3.2 Fatigue tests

The fatigue tests were performed with constant load amplitudes and a frequency of approx. 30 Hz. A high stress ratio $R = 0.5$ was chosen according to the IIW Fatigue Design Recommendations (Hobbacher, 2009). One or two load levels were selected for the tests of each series of specimens. All specimens failed due to root cracking. The failure criterion for terminating the tests was a through-thickness crack. After this, the remaining life of the specimens was very short.

The test results, which are further described by Fricke et al. (2006), are shown in Fig. 4.15 in terms of the nominal stress range in the RHS member at the weld. The S-N curves given for a probability of survival of 50% and 97.7% were obtained from statistical analysis assuming a fixed slope exponent $m = 3$.

The results for the bending load show that the welding process affects the fatigue life, which is probably due to differences in the sharpness of the weld root.

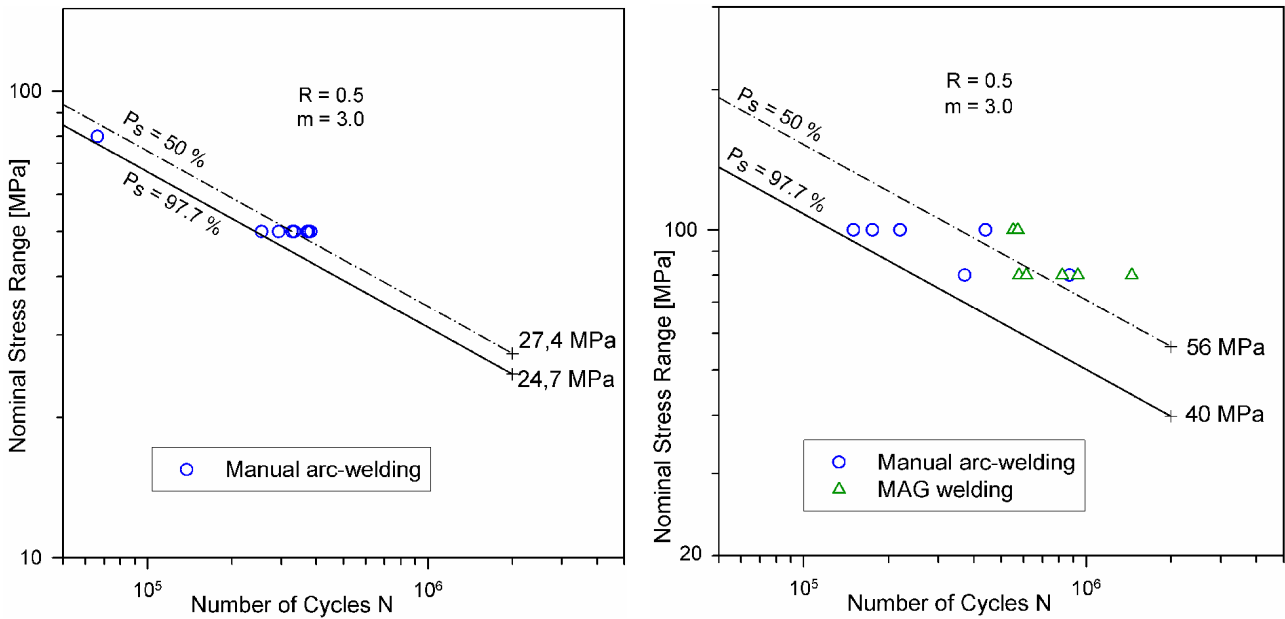


Fig. 4.15: Fatigue lives obtained for axial load (left) and bending load (right)

4.3.3 Application of the nominal stress approach

In the IIW Fatigue Design Recommendations (Hobbacher, 2009), this detail is classified as FAT 45 (structural detail no. 424). It should be kept in mind that the nominal stress for root failure is referred to the weld throat area.

When converting the nominal stress in the 6 mm thick RHS wall to the weld throat thickness of $a = 4.3$ mm and 5.1 mm, the tests under axial load show a fatigue class of just below FAT 36, whereas the bending tests yield a fatigue class of over FAT 45. The difference can be explained by the effect of local weld throat bending. Under axial load, the fatigue cracks appear at the long side of the RHS-joint, which is less restrained to local bending, whereas during bending they appear at the short upper side of the joint, which is the most highly stressed one and where local bending is more restrained.

The same reason might be responsible for the non-conservative fatigue assessment under axial load. Local approaches should be able to consider these effects.

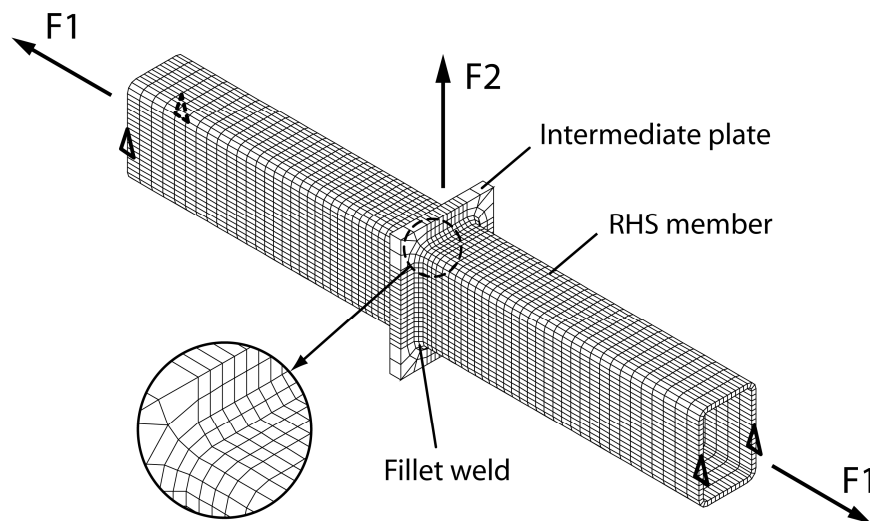


Fig. 4.16: Finite element model and loading of the RHS-joint

4.3.4 Application of the structural stress approach using solid elements

The numerical analysis of the structural stress was performed with the finite element model shown in Fig. 4.16, using 20-node solid elements. Only one element was arranged over the wall and weld thickness. The axial loading is realised by the forces F_1 , while the force F_2 creates the 3-point bending.

The structural weld stress was analysed in the weld leg section according to Fig. 3.6 using the nodal forces. The last column of Table 4.5 shows the computed stress concentration factors $K_{s,w}$, i. e. the ratio between structural weld stress and nominal stress in the RHS-member. The stress concentration factors clearly show the effect of increased local bending under axial load mentioned above. Furthermore it can be seen that the weld throat thickness influences also the stress concentration factor, which explains the longer fatigue lives of the MAG-welded specimens as plotted in Fig. 4.15.

Table 4.5: Computed structural stress concentration factors $K_{s,w}$

Specimen type	Loading type	a	$K_{s,w}$
RHS, manual metal-arc welding	axial	4.3 mm	4.25
RHS, manual metal-arc welding	bending	4.3 mm	3.37
RHS, MAG-welded	bending	5.1 mm	2.78

Fig. 4.17 shows the test results in an S-N diagram lying well above the design S-N curve FAT 80 based on structural stress ranges mentioned earlier. It can be seen from Fig. 4.17 that the results for axial load are in the lower part of the scatter band, while those for bending load are in the upper part. This might be due to the more localised stress concentration under bending, resulting in semi-elliptical cracks showing slower crack propagation.

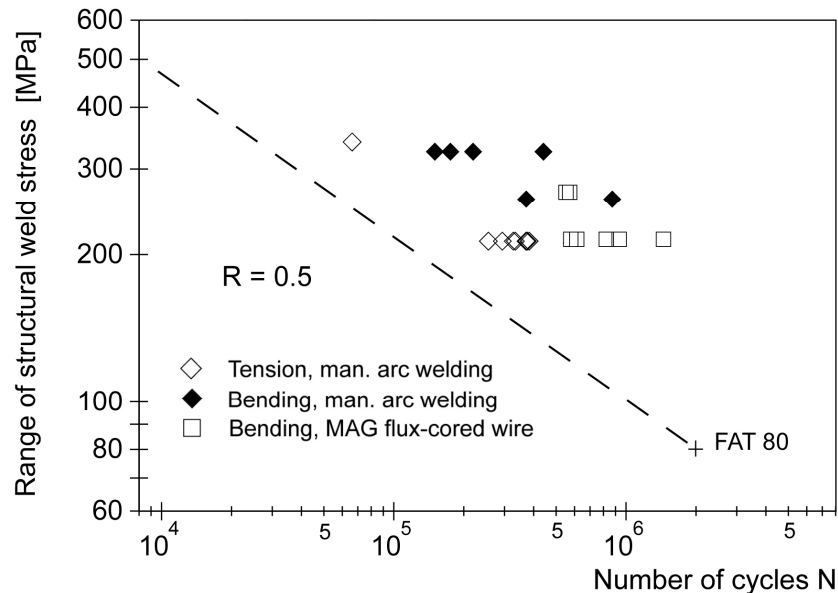


Fig. 4.17: S-N results of the fatigue tests using the computed range of structural weld stress

4.3.5 Application of the structural stress approach using shell elements

A corresponding finite element modeling is performed using shell elements and the technique described in Fig. 3.7. The structural stress is computed at the weld leg section according to equations (3.11), (3.14) and (3.15). Fig 4.18 gives the distribution of the structural stress concentration factor

along the weld line. For the axial load, maximum $K_{s,w}$ is obtained at mid height of the RHS member and for the bending load, it is located at mid distance of the upper weld seam, which corresponds to the crack locations in the tests.

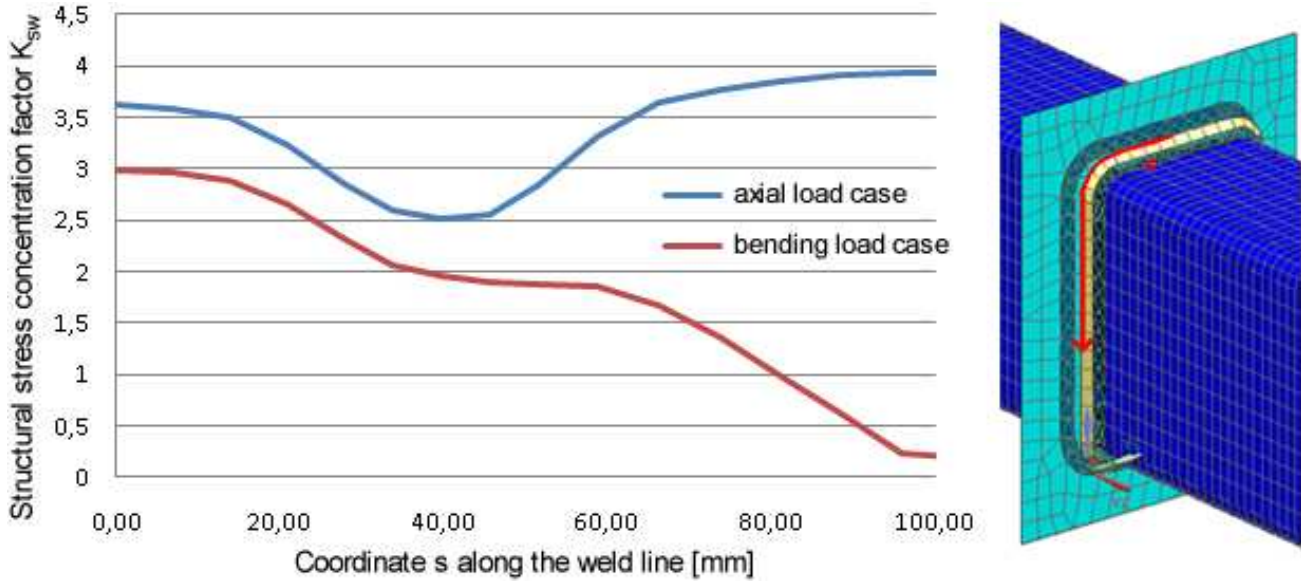


Fig. 4.16: Distribution of structural stress concentration factor $K_{s,w}$ along the weld line for a weld throat thickness $a = 4.3$ mm

The maximum stress concentration factors $K_{s,w}$ are given Table 4.6. These are about 10% smaller than those calculated with the solid model (see Table 4.5). However, also with these factors the test results would be above the design S-N curve acc. to FAT 80 which can be concluded from Fig. 4.15 based on the solid finite element model. The last columns in Table 4.6 show the bending portion of the structural weld stress and the shear stress in the weld leg section, which is rather small so that the approach is applicable.

Table 4.6: Computed structural stress concentration factors $K_{s,w}$ and stress components

Specimen type	Loading type	a	$K_{s,w}$	(σ_b/σ_s)	(τ/σ_s)
Manual metal-arc welding	axial	4.3 mm	3.92	79.8%	0.55%
Manual metal-arc welding	bending	4.3 mm	2.99	75.5%	1.8%
MAG-welded	bending	5.1 mm	2.54	75.7%	1.9%

4.3.6 Application of the effective notch stress approach

Several computations with the effective notch stress approach were performed in a round robin for the IIW Guideline on this approach (Fricke, 2008). One representative analysis is described here, using the actual mean weld throat thicknesses given above (Fricke and Kahl, 2006).

The computation of the notch stresses was performed using the submodel technique. The coarse model shown in Fig. 4.16 was slightly refined in order to obtain improved prescribed displacements for the submodels. These were created with very fine meshes and subjected to prescribed deformations at the boundaries taken from the coarse model. Fig. 4.19 shows typical submodels.

The weld root was modelled with a keyhole shape and a reference notch radius $r_{ref} = 1$ mm. 8-noded solid elements were chosen, having a length of about 0.1 mm at the notch.

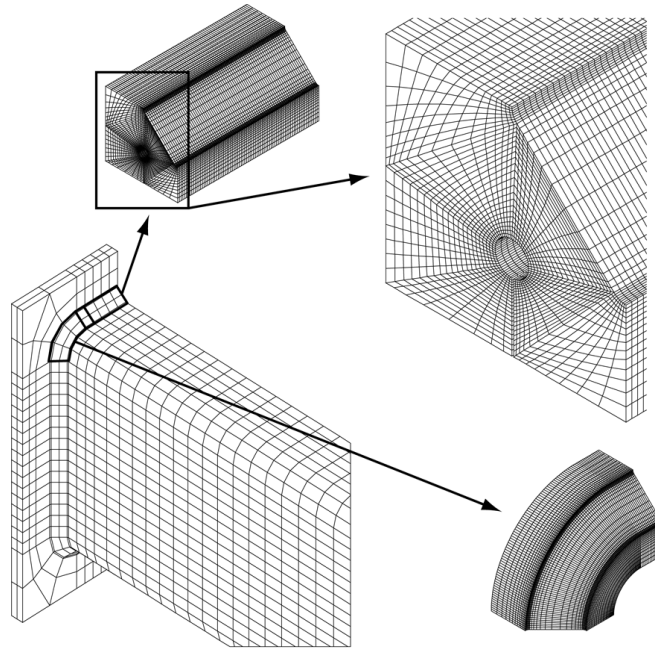


Fig. 4.19: Coarse finite element model and submodels for the notch stress analysis

To account for the different weld throat thicknesses given above, two models have been created. The computed fatigue notch factor K_f along the weld root line, related to the nominal stress in the RHS, is shown in Fig. 4.20 with a max. value $K_f = 9.03$ for the axial load case and $K_f = 6.50$ for the bending load case. The latter is decreased to $K_f = 5.97$ if the throat thickness is increased to $a = 5.1$ mm for the MAG welded joints. The distribution differs slightly from that in Fig. 4.18.

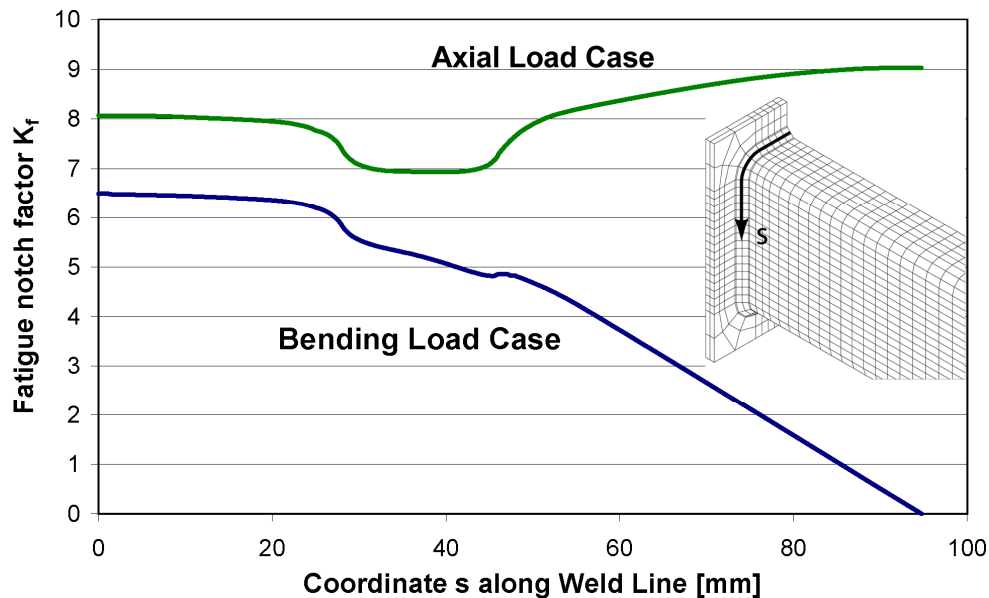


Fig. 4.20: Distribution of the fatigue notch factor along the weld root line for a weld throat thickness $a = 4.3$ mm

Based on these factors, the fatigue lives are plotted in Fig. 4.21 in relation to the effective notch stress range. It can be seen that all test results are located above the S-N curve for the fatigue class FAT 225 according to the IIW Fatigue Design Recommendations (Hobbacher, 2009). Again, the values for bending are above those for axial loading as discussed in the previous sub-chapters.

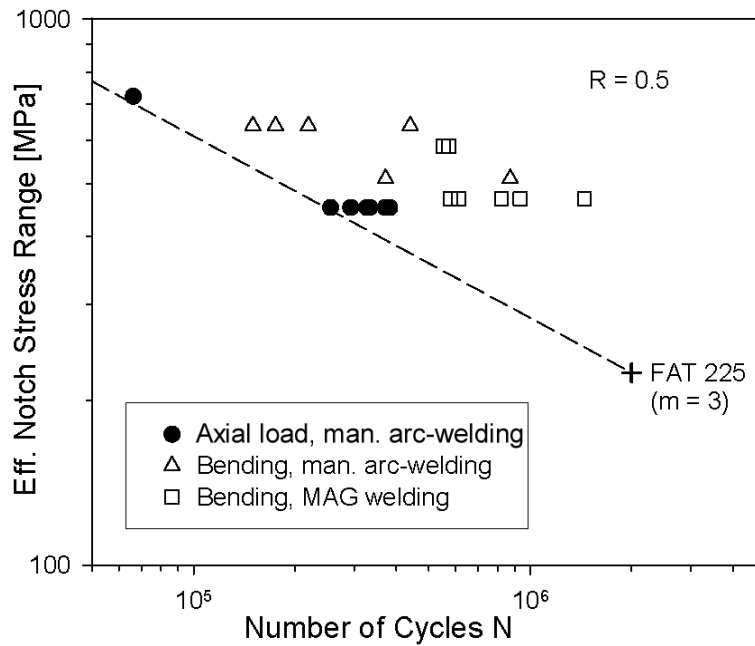


Fig. 4.19: S-N results of the fatigue tests using the computed range of effective notch stress

4.4 Lap Joints and Cover Plates

4.4.1 Description of the detail

The fourth detail concerns test specimens of *lap joints* with an interrupted main plate and *cover plates* on a continuous main plate. In the first case, the fillet welds are full-load carrying, whereas those at cover plates are partial-load carrying, which has an effect on the local stresses at the weld toe and root. Fig. 4.22 shows the geometry of the specimens. The arrangement of plates on both sides should avoid local bending. A plate thickness of 12 mm and two different weld throat thicknesses of $a = 3$ and 7 mm were chosen resulting in four different types of specimens, denoted L.3, L.7, C.3 and C.7.

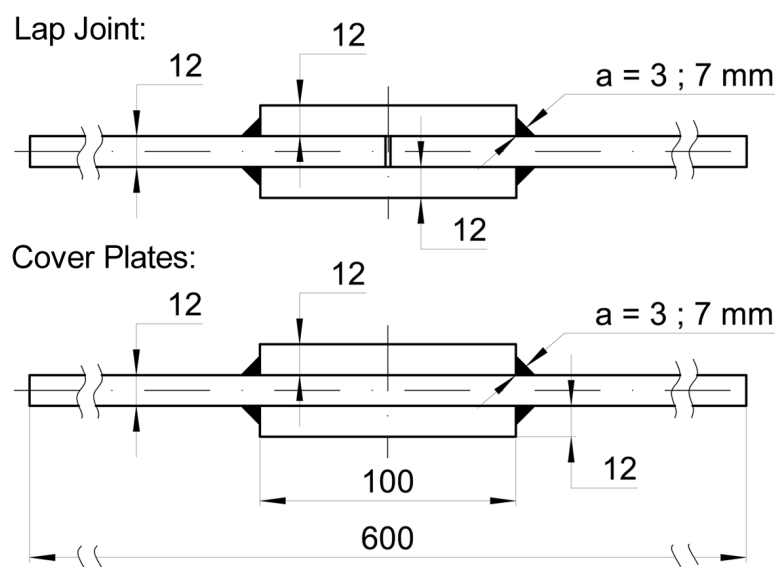


Fig. 4.22: Geometry of the lap joints and cover plates

The test specimens were fabricated from structural steel S355. While the fillet welds with $a = 3$ mm were performed with one run, 3 runs were necessary for the throat thickness of 7 mm. The actual thickness varied between 2.9 and locally 5.0 mm and between 6.6 and 7.6 mm, respectively. In total, 10 specimens were fabricated for each geometry. Further details are described by Fricke and Feltz (2010).

4.4.2 Fatigue tests

The fatigue tests were performed with constant load amplitudes at different load levels, corresponding to nominal stress ranges in the plate in front of the weld between $\Delta\sigma_n = 90$ and 210 N/mm^2 . The stress ratio was $R \approx 0$.

Depending on the type of specimen, fatigue crack initiation was observed either at one of the weld toes, running through the main plate, or at one of the weld roots, propagating through the weld throat. Fracture of the specimen was taken as failure criterion as usual for small-scale specimens. Mainly weld toe failures occurred, while weld root cracks were observed in all lap joints with a weld throat thickness $a = 3$ mm (L.3) and three cover plates with the same weld throat thickness (C.3).

The fatigue lives of each test series were statistically evaluated assuming forced slope exponent $m = 3$ for the S-N curves being typical for welded joints.

Fig. 4.23 shows the fatigue lives of all specimens with weld toe failures. All results seem to fall into one scatter band. It is interesting to note that no difference is recognizable between lap joints (full-load carrying) and cover plates (partial-load carrying). The evaluated characteristic fatigue strength for a survival probability $P_s = 97.7\%$ at $N = 2 \cdot 10^6$ cycles is 78 N/mm^2 .

The fatigue lives of the lap joints with throat thickness $a = 3$ mm showing root failures (L.3) are summarized in Fig. 4.24. The statistical evaluation yields a characteristic fatigue strength of 40 N/mm^2 . It should be kept in mind that the nominal stress is based on the main plate and that the weld stress is twice as high. The S-N curve seems to have a shallower slope causing a wide scatter band when $m = 3$ is assumed.

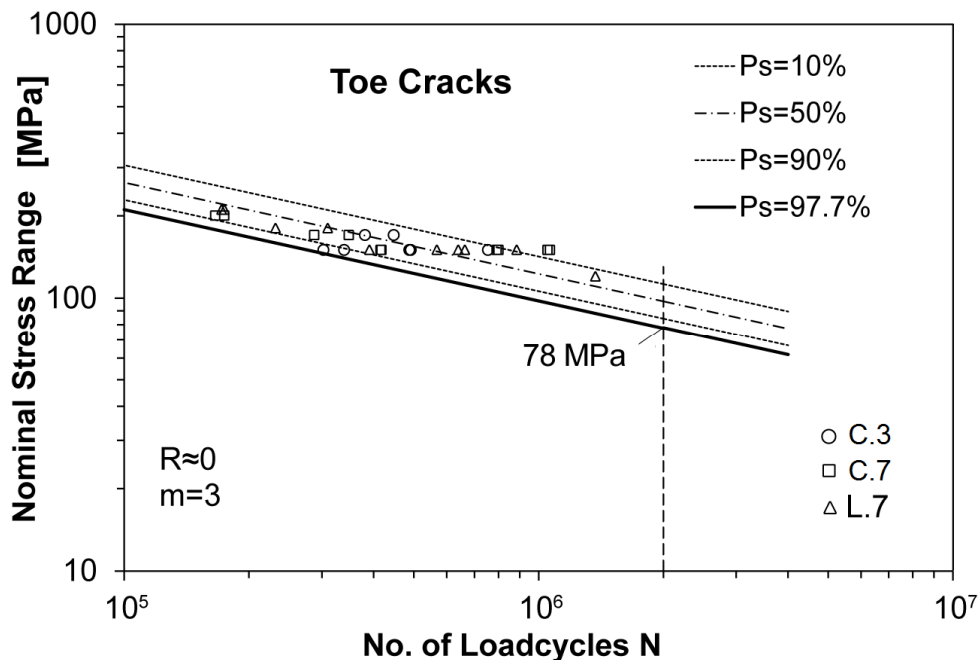


Fig. 4.23: S-N results for all specimens with weld toe failure

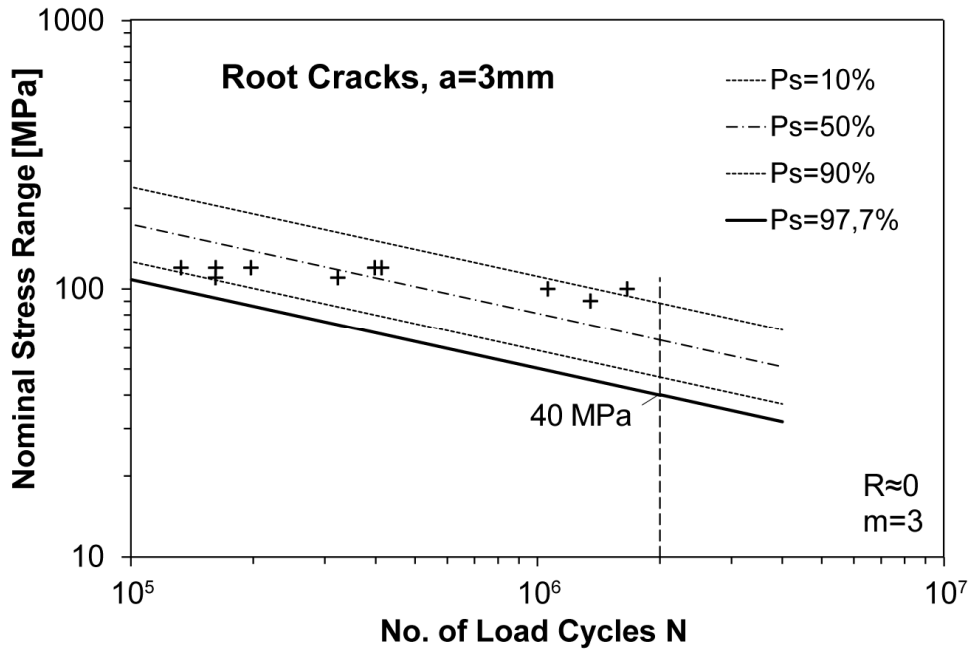


Fig. 4.24: S-N results for all L.3 specimens with weld root failure

4.4.3 Application of the nominal stress approach

The details investigated can be found in the catalogues of codes and recommendations based on the nominal stress approach. In the case 611 of the IIW Fatigue Recommendations (Hobbacher, 2009), the *lap joint* is classified as FAT 63 for weld toe failure (based on nominal stress in main plate) and FAT 45 for weld throat failure (based on the nominal stress in weld throat). The above mentioned test results are generally above these values, considering the difference between plate and weld stress.

The FAT value of the *cover plate* in the recommendations depends on the structural configuration. On a continuous plate it is classified as FAT 71 (case 513), whereas on an I-beam or a rectangular hollow section it corresponds to FAT 50 (cases 711 and 713). The first case agrees better with the test specimen because the latter includes additional structural stress concentrations. The test results are again on the safe side.

4.4.4 Application of the effective notch stress approach

The effective notch stress approach, using the elastic stress in the notches rounded by a reference radius $r_{ref} = 1$ mm, is able to assess both weld toes and weld roots. Relatively fine-meshed finite element models have been created for the numerical analyses (Fig. 4.25) using plane strain conditions and considering the recommendations by Fricke (2008). The keyhole notch has been placed such that the minimum distance between the rounded notch surface and the weld surface is exactly the throat thickness (3 mm or 7 mm, respectively).

The analysis has been performed with contact elements between the main and the lap/cover plate. A nominal stress of $\sigma_n = 150$ N/mm² has been applied being typical for the tests.

The resulting stress concentration factors, i. e. the max. principal stress in the notch related to the nominal stress, are given in Table 4.7. The expected failure location agrees with the tests only for two specimen types (L.3 and C.7). The notch stress at the weld root seems to be unrealistically high for the lap joint with $a = 7$ mm (L.7). It can be stated that the effective notch stress is quite sensitive to the weld throat thickness, but also to the load carrying grade, particularly at the weld root.

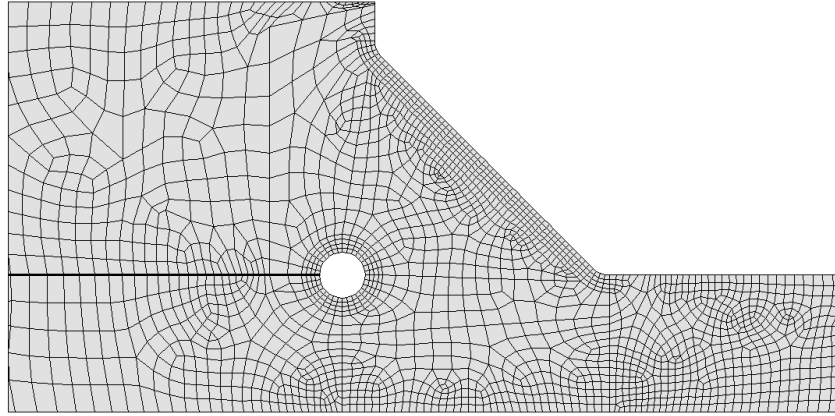


Fig. 4.25: Typical mesh (quarter model) for the effective notch stress analysis

Table 4.7: Computed notch stress concentration factors for reference radius $r_{ref} = 1$ mm (expected failure location highlighted)

Specimen type	K_f	
	at toe	at root
Lap joint $a=3$ mm	5.76	7.34
Cover plate $a=3$ mm	3.54	3.60
Lap joint $a=7$ mm	3,35	4.12
Cover plate $a=7$ mm	2,82	2.24

Fig. 4.26 shows the fatigue test data, using the effective notch stresses for the actual crack initiation site. It can be seen that the data are generally above the S-N curve according to FAT 225, however, the results for the root cracks are quite conservative which is obviously due to the increased stress concentration by the keyhole notch.

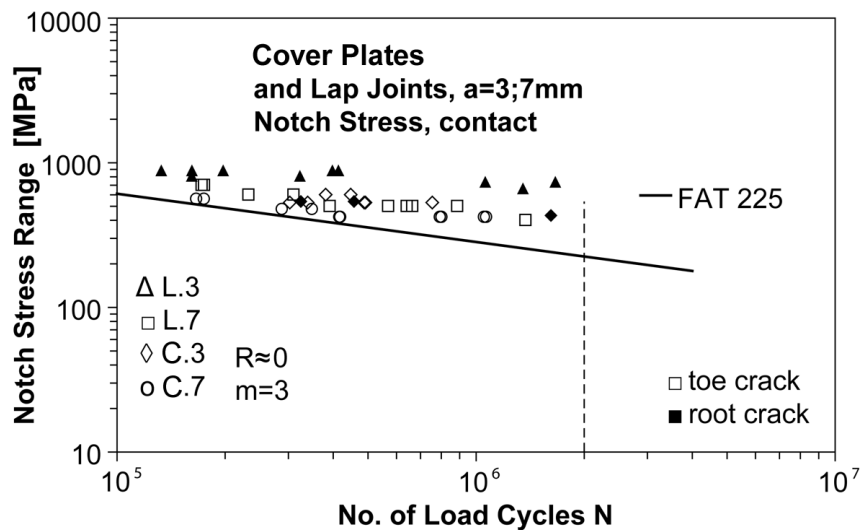


Fig. 4.26: S-N data based on the effective notch stress

4.4.5 Application of the strain energy density (SED) approach

2D finite element models of the four types of specimens have been created, using plane strain elements with quadratic shape function. Contact elements were again applied to the slit between the

main and lap/cover plate. Fine meshes as shown in Fig. 3.16 have been created where the average SED was determined for the control sector with $R_0 = 0.28$ mm using a subroutine in ANSYS (Fischer et al., 2010). In addition, relatively coarse meshes with an element length of $\ell = 0.28$ mm and 1.0 mm at the notch tip were used. The SED of the latter has been converted from $R^* = 1$ mm to $R_0 = 0.28$ mm. The computed range of the SED, $\Delta \bar{W}$, is given in Table 4.8 again for a nominal stress range $\Delta \sigma_n = 150$ N/mm².

Table 4.8: Computed $\Delta \bar{W}$ from different meshes for $R_0 = 0.28$ mm and $\Delta \sigma_n = 150$ N/mm² (deviations in % refer to fine mesh results; expected failure location highlighted)

	$\Delta \bar{W}$ [Nmm/mm ³] for weld toe			$\Delta \bar{W}$ [Nmm/mm ³] for weld root*			σ_T^{**} [MPa]
	fine mesh (0.03mm)	coarse mesh (0.28mm)	coarse mesh (1mm)	fine mesh (0.03mm)	coarse mesh (0.28mm)	coarse mesh (1mm)	
L.3	0.725	0.724 (0.2%)	0.686 (5.4%)	1.107	1.089 (1.7%)	1.076 (2.8%)	113
C.3	0.352	0.352 (0.1%)	0.340 (3.4%)	0.236	0.232 (1.6%)	0.225 (4.5%)	99
L.7	0.317	0.319 (0.7%)	0.315 (0.4%)	0.393	0.395 (0.6%)	0.382 (2.9%)	83
C.7	0.241	0.244 (0.9%)	0.244 (1.1%)	0.072	0.074 (2.3%)	0.0678 (9%)	83

*) smaller values would be obtained for $R_0 = 0.36$ mm

**) T-stress used for the conversion of $\Delta \bar{W}$ from $R = 1$ mm to $R_0 = 0.28$ mm

The difference between the coarse and fine meshes is small as long as the element length ℓ does not exceed 0.28 mm. For the coarse meshes with $\ell = 1$ mm, the deviations are larger, but tolerable. At the weld root the effect of the T-stress has been considered as described above (eq. 3.20).

The largest SED-value at the toe or root determines the expected crack initiation site. It can be seen from Table 4.8 that the derived crack initiation site corresponds to the tests for three of the four types of specimens. The lap joint with $a = 7$ mm (L.7) shows the highest SED at the weld root, however, fatigue cracks occurred at the weld toe. It should be mentioned that the SED at the weld root would be smaller if the control radius would be $R_0 = 0.36$ mm instead of 0.28 mm. This would change the crack initiation site.

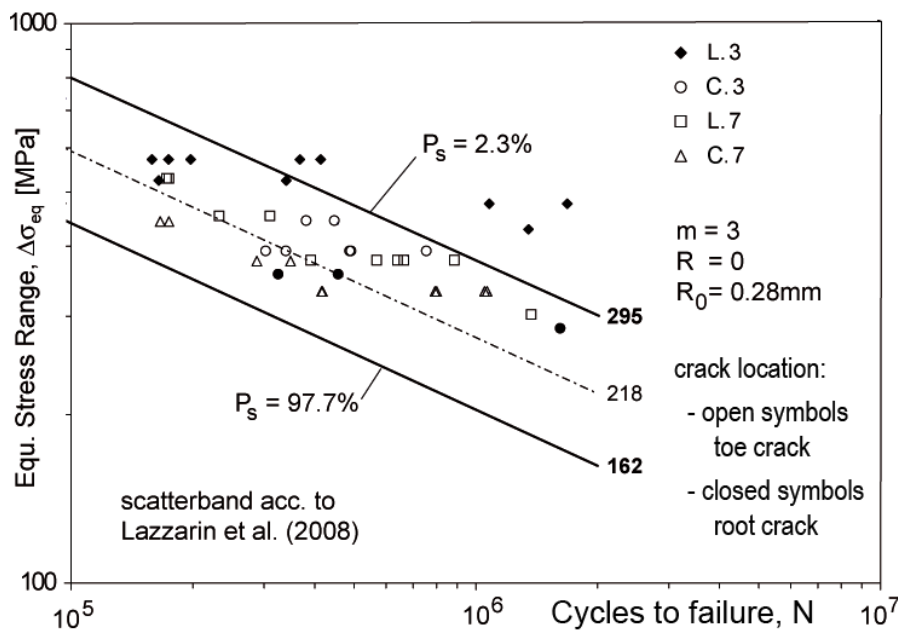


Fig. 4.27: S-N diagram showing the test results in terms of the equivalent stress

The average SED has been computed for different load levels because the contact creates a slight non-linear system with increasing load. This allows plotting the results in the S-N diagram in Fig. 4.27, here using the equivalent stress range according to eq. (3.17) on the vertical axis instead of the SED. Also included is the scatter band according to the evaluation in Fig. 3.14, here in terms of the equivalent stress range.

Most of the results are within the scatter band except for the series L.3 failing from the weld root. The shallow slope of the S-N curve based on nominal stresses (Fig. 4.24) can be recognized also here. The results are on the safe side - as in the nominal and notch stress approaches.

4.4.6 Application of the crack propagation approach

The crack propagation analysis has been performed for the four types of specimens using the program FRANC2D (2004) which applies the finite element method to the computation of the stress intensity factor (SIF). Using a relatively fine two-dimensional basic mesh, the area around the crack tip is re-meshed with appropriate fineness containing the crack. The SIF is calculated with the J integral method. This is performed for various crack lengths with given increment Δa , while the direction of crack propagation is determined by the SIFs for mode I and mode II. The finite element model in FRANC2D does not contain contact elements. Therefore, the contact between the main and cover or lap plate was realized by additional elements in the gap having usual axial stiffness, but negligible shear stiffness.

The fatigue life is determined using the computed SIFs and integrating the Paris-Erdogan law numerically from the initial to the final crack length. The initial crack length a_i has been assumed as 0.1 mm, whereas the final crack length has been set to approximately half plate thickness. The remaining life is neglected. The material parameters $m = 3$ and $C = 5.21 \cdot 10^{-13}$ (units: N and mm) recommended by Hobbacher (2009) have been used, representing upper limit of the scatter band (2.3% probability of survival).

Fig. 4.28 shows typical crack paths computed for the weld toe and the weld root. While the crack at the weld toe follows approximately the direction along half the notch opening angle, the direction is almost vertical at the weld root which agrees with the observations in the tests at least in the middle part of the specimens.

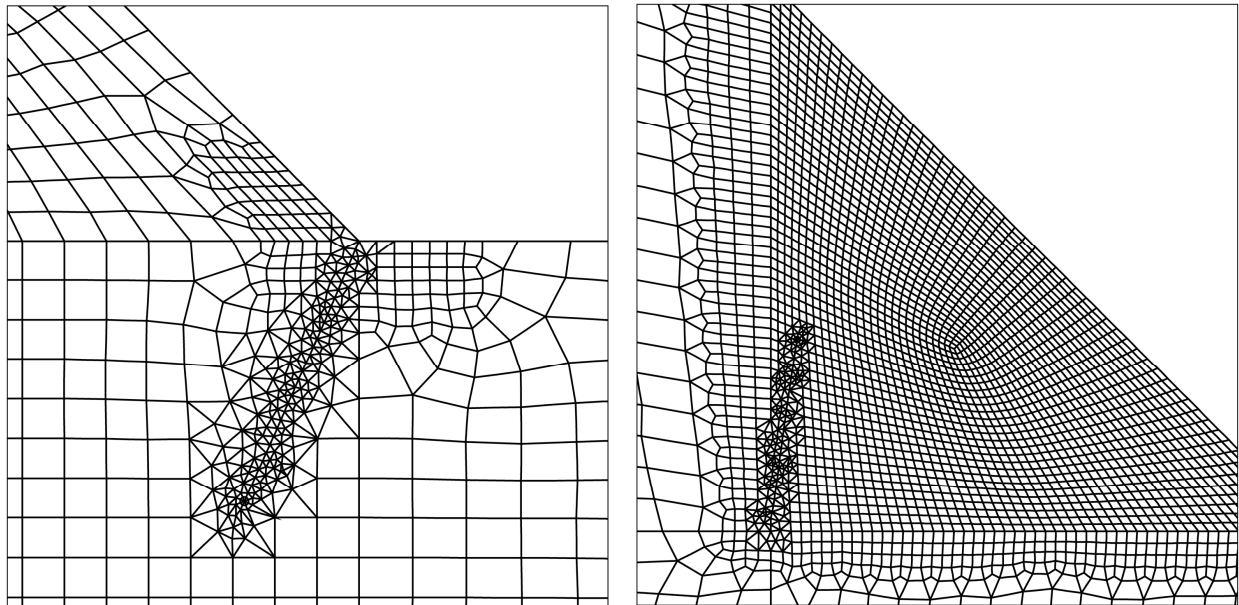


Fig. 4.28: Typical crack paths at the weld toe (left) and weld root (right)

The fatigue life (number of load cycles N) has been determined again with a nominal stress range $\Delta\sigma_n = 150 \text{ N/mm}^2$ in the main plate again. From this, a characteristic fatigue strength $\Delta\sigma_c$ based on

nominal stress was determined by extrapolating the result with a slope exponent $m = 3$ to $N = 2 \cdot 10^6$. Table 4.9 summarizes the results.

Table 4.9: Computed fatigue life N from crack propagation approach for $\Delta\sigma_n = 150 \text{ N/mm}^2$ and derived characteristic fatigue strength $\Delta\sigma_c$ (expected failure location highlighted)

	fatigue life N , crack at:		$\Delta\sigma_c [\text{N/mm}^2]$, crack at:	
	weld toe	weld root	weld toe	weld root
L.3	15,924	7,112	30.0	22.9
C.3	40,835	52440	41.0	44.6
L.7	49,313	62,620	43.5	47.3
C.7	67,513	406,617	48.5	88.2

4.4.7 Comparison of all results on the basis of nominal stresses

The results from the different approaches can be compared on the basis of the characteristic fatigue strength at $2 \cdot 10^6$ cycles for a survival probability $P_s = 97.7\%$ based on nominal stress.

Table 4.10 summarizes all computed characteristic fatigue strengths together with those derived from the tests. The relatively simple nominal stress approach is only partly able to identify the crack initiation site. The results for the thin welds (L.3 and C.3) show that the fatigue critical locations are generally quite well identified if the small difference between the values for the effective notch stress of C.3 is neglected. The same applies also to the cover plates with large weld throat thickness (C.7). Only the fatigue-critical location for the lap joint with thick welds (L.7; toe failure) is not well predicted. In the SED approach this might be due to the too small control radius, whereas the results of the effective notch stress approach seem to be affected by the keyhole notch creating a too severe notch effect.

The crack propagation approach predicts the critical crack location very well, but the results are generally too low in comparison with those found in the tests (Figs. 4.23 and 4.24). A reason might be that the 2D analysis is rather conservative by assuming a continuous crack along the weld toe or root line, whereas the first part of crack propagation might be governed by semi-elliptical crack shapes showing a reduced crack propagation rate. In contrast to the first example, semi-elliptical cracks are reasonable also for the weld root where the slit is no more perpendicular but parallel to the stresses. When assuming semi-elliptical crack shapes at both locations, the relation in fatigue lives between weld toe and weld root may remain unchanged.

Table 4.10: Characteristic fatigue strengths $[\text{N/mm}^2]$ based on nominal stresses in main plate and $m = 3$ (expected failure location highlighted)

Specimen type:	L.3		C.3		L.7		C.7	
	toe	root	toe	root	toe	root	toe	root
Crack location:								
Fatigue test (as-welded)	--	40	78	--	78	--	78	--
Nominal stress approach	63	22.5	71	--	63	53	71	--
Effect. notch stress approach	39.1	30.7	63.5	62.5	67.2	54.6	79.8	100.4
SED Approach*	42.4	34.3	60.9	74.4	64.2	57.6	73.5	134.6
Crack propagation approach	30.0	22.9	41.0	44.6	43.5	47.3	48.5	88.2

*) computed with $R_0 = 0.28 \text{ mm}$

4.5 Fillet-Welded Pipe Penetration with Multiaxial Loading

4.5.1 Description of the detail

The fifth detail is a pipe penetration through a plate connected with fillet welds, see Fig. 4.29. The plate is subjected to an axial nominal stress σ_n , creating a complex load situation in the weld. Weld root cracking is here possible particularly at the 45° -position, where radial stresses σ_r and shear stresses τ are transferred by the fillet welds. These stress components shall be determined and the weld stress be superimposed according to eq. (3.6).

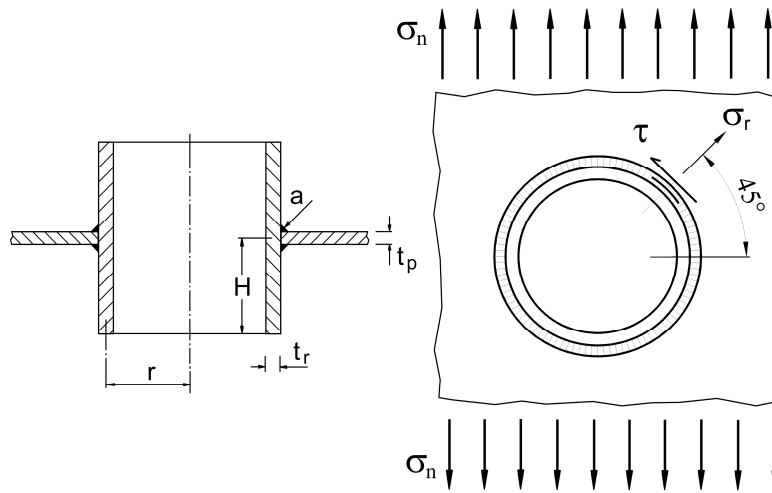


Fig. 4.29: Geometry of the pipe penetration and location of the point considered

The data of the pipe penetration and loading are as follows:

Plate thickness	t_p	=	20 mm
Tubular thickness	t_r	=	30 mm
Mean radius	r	=	400 mm
Half height of tube	H	=	150 mm
Weld throat thickness	a	=	8 mm
Stress range in plate	$\Delta\sigma$	=	200 N/mm ²

4.5.2 Application of the nominal stress approach using stress concentration factors

The normal and shear stress ranges $\Delta\sigma_r$ and $\Delta\tau$ in the plate beneath the weld can be determined using the design charts in DNV (2010). These show the structural stress concentration factor K_s (there termed SCF) for these two stress components at different locations around the pipe as function of the following geometry parameters:

$$t_r/t_p (= 1.5)$$

$$r/t_p (= 20)$$

$$H/t_r (= 5)$$

Fig. 4.30 shows the design charts for the normal and shear stresses at 45° for $H/t_r = 5$, from which stress concentration factors $K_s = 0.325$ and 0.75 can be read off, resulting in the following stress components in the plate:

$$\Delta\sigma_r = 65 \text{ N/mm}^2 \quad \text{and} \quad \Delta\tau = 150 \text{ N/mm}^2$$

The corresponding stresses in the weld can be calculated according to Fig. 3.2 as follows:

$$\Delta\sigma_{\perp} = \Delta\tau_{\perp} = \frac{\Delta\sigma_r \cdot t_p}{2a\sqrt{2}} = 57.4 \text{ N/mm}^2 \quad (4.2)$$

$$\Delta\tau_{\parallel} = \frac{\Delta\tau \cdot t_p}{2a} = 187.5 \text{ N/mm}^2 \quad (4.3)$$

The following nominal weld stress results from eq. (3.6):

$$\sigma_{n,w} = \sqrt{\sigma_{\perp}^2 + \tau_{\perp}^2 + 0.2 \cdot \tau_{\parallel}^2} = 116.7 \text{ N/mm}^2 \quad (4.4)$$

The fatigue assessment can be performed on the basis of the fatigue class FAT 36 as $a > t_p/3$. Comparisons with different fatigue tests on similar geometries have shown the suitability of the procedure (Lotsberg, 2004).

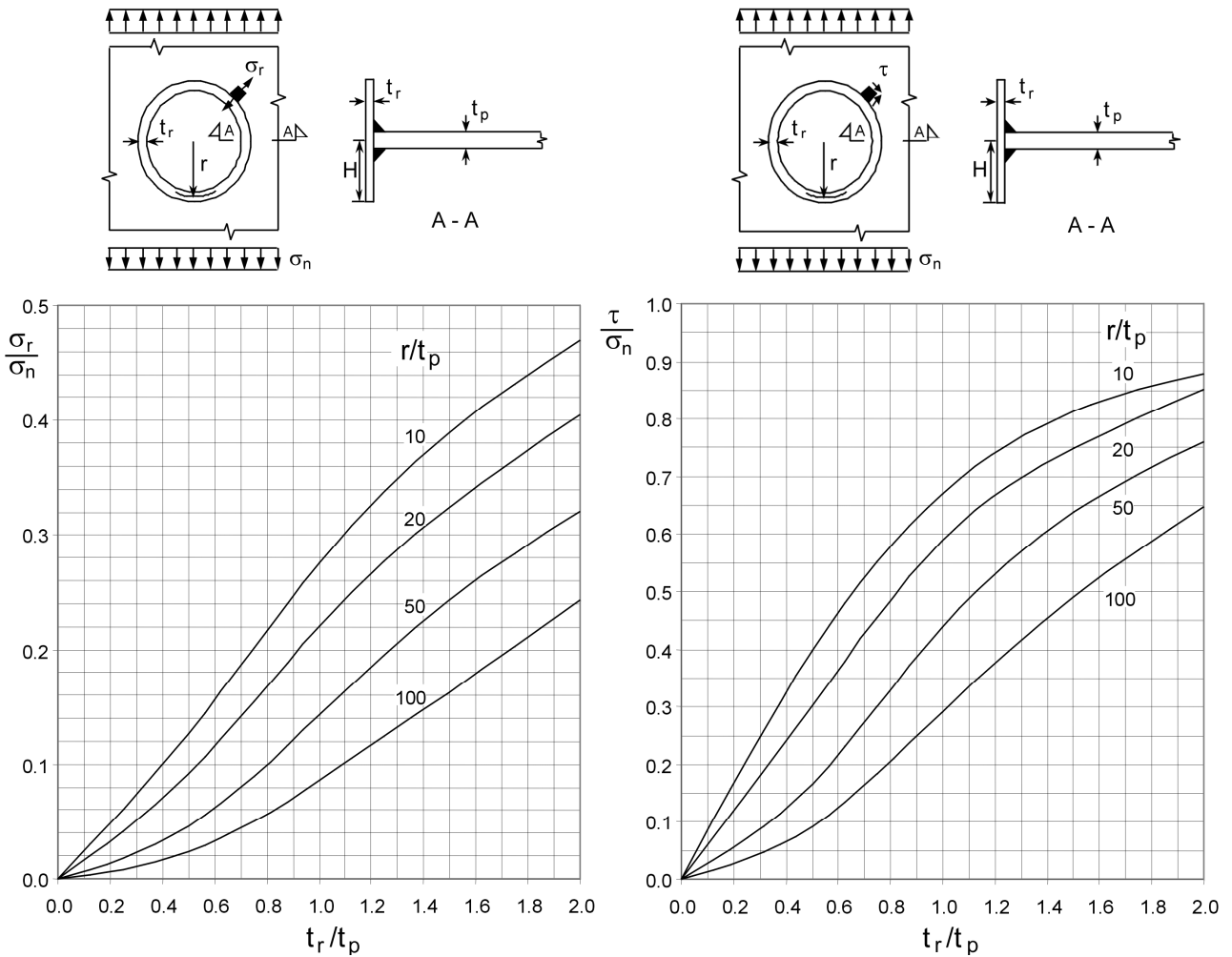


Fig. 4.30: Design charts for the normal and shear stress in the plate at 45° for $H/t_r = 5$ (DNV, 2010)

4.5.3 Application of the nominal stress approach using the finite element method

The model is created using shell elements in the way proposed in Fig. 3.7, with the geometrical parameters described above. The size of the plate is 5000 x 5000 mm (see Fig. 4.31) which can be

assumed as infinite relative to the size of the tube. Tensile load which corresponds to the nominal stress amplitude of 200 N/mm^2 is applied.

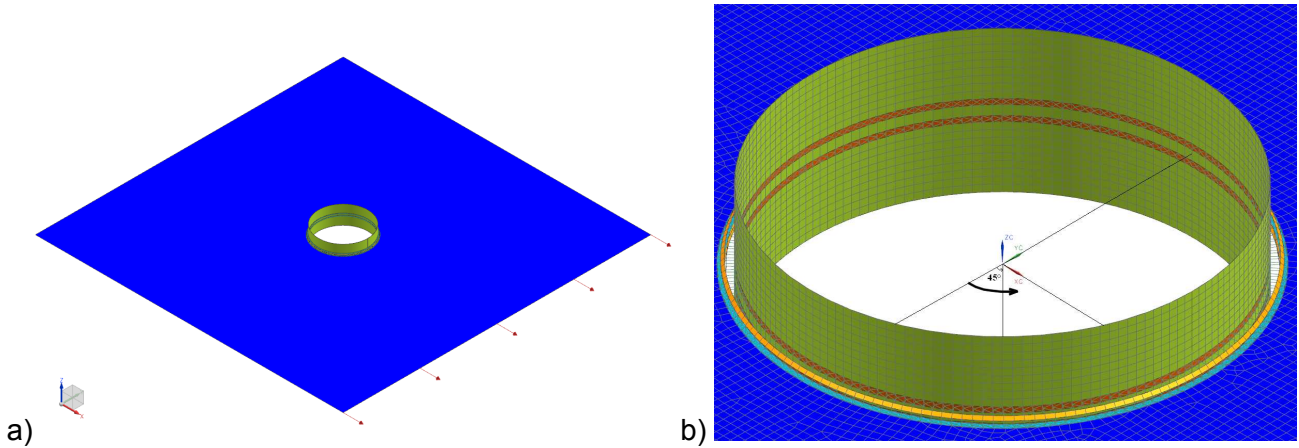


Fig. 4.31: Finite element model and angle definition

To evaluate the multiaxial nominal stresses in the weld, internal forces are extracted along the weld leg line. The nominal stress components in the weld $\Delta\sigma_{\perp}$, $\Delta\tau_{\perp}$ and $\Delta\tau_{\parallel}$ are calculated directly from the forces using the weld throat thickness a . The three stress components and the resultant nominal weld stress $\Delta\sigma_{n,w}$ acc. to eq. (3.6) for the angle of 45° are found as follows:

$$\Delta\sigma_{\perp} = 57.3 \text{ N/mm}^2; \Delta\tau_{\perp} = 72.3 \text{ N/mm}^2; \Delta\tau_{\parallel} = 168.6 \text{ N/mm}^2; \Delta\sigma_{n,w} = 119.2 \text{ N/mm}^2$$

The results are very close to the values in the previous section. The fatigue assessment is performed with the fatigue class FAT36 as mentioned before.

4.5.4 Application of the effective notch stress approach

The effective notch stress was computed with the fine-mesh finite element model shown in Fig. 4.32 representing 1/8 of the structure by utilizing symmetry conditions of three planes including that at half plate thickness. The plate width chosen corresponds to ten times the pipe diameter in order to eliminate any boundary effects on the results.

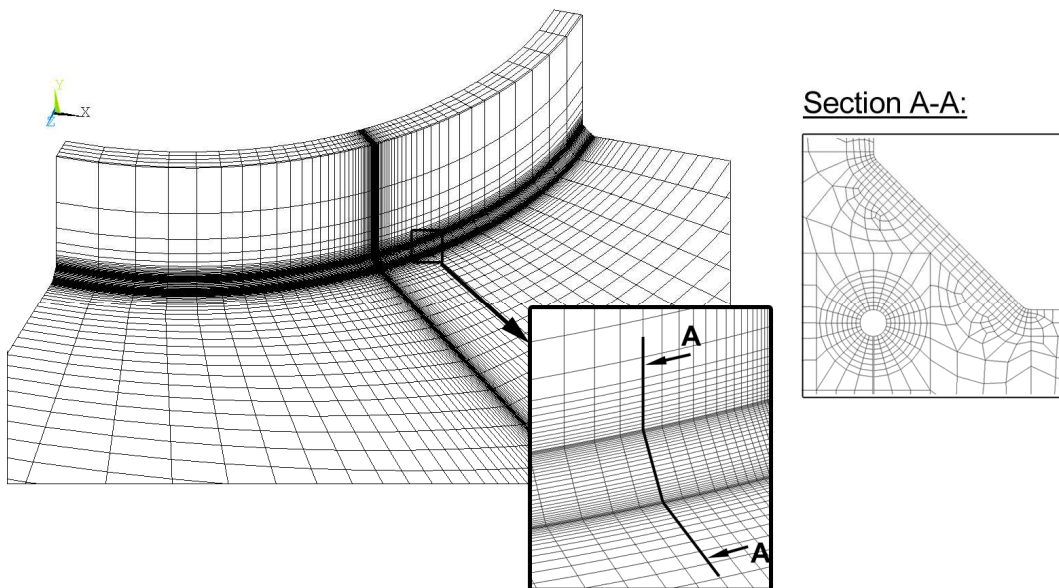


Fig. 4.32: Finite element model of the pipe penetration for the application of the effective notch stress approach

Solid elements with quadratic shape function were applied. The weld root was idealized with a keyhole shape and element lengths down to 0.26 mm.

The nominal stress range $\Delta\sigma_n = 200 \text{ N/mm}^2$ creates in the rounded notch surface at the 45° position the following principal stress components and equivalent von Mises stress range:

$$\Delta\sigma_1 = 740 \text{ N/mm}^2; \Delta\sigma_2 = -209 \text{ N/mm}^2; \Delta\sigma_{eq} = 853.5 \text{ N/mm}^2$$

The results show that here we have a typical case with pre-dominant shear stress, which is characterized by principal stresses with different signs, see chapter 3.4.3. The fatigue assessment is recommended to be based on the equivalent von Mises stress together with a reduced fatigue class, i. e. FAT 200.

4.5.5 Comparison of the results

A comparison between all results can be performed on the basis of the characteristic fatigue strength $\Delta\sigma_c$ based on nominal stress which can be derived from the assumed nominal stress, the computed local stress and the FAT class of the S-N curve. The results in Table 4.11 show some significant differences between the results. The relatively low fatigue strength obtained with the notch stress approach might be due to the thickness effect acting here.

Table 4.11: Characteristic nominal fatigue strength derived from the approaches

Characteristic fatigue strength $\Delta\sigma_c$ based on nominal stress, derived from	
nominal stress approach	effective notch stress approach
60.4 - 61.7 N/mm ²	46.9 N/mm ²

4.6 Laser-Stake Welded T-Joint

4.6.1 Description of the detail

The investigation of laser-stake welded T-joints was reported by Frank et al. (2011), where 8 mm thick plates were used in three of four series. Fig. 4.33 shows the geometry. The web plate of the joint was subjected to cyclic tensile stress $\Delta\sigma_n$, while the face plate was clamped by bolts. Fatigue cracks occurred at the laser-welded connection between web and face plate.

The S-N curve derived from all fatigue tests showed a mean fatigue strength of 48 N/mm^2 at $2 \cdot 10^6$ load cycles with a slope exponent of $m = 4.5$. If a typical scatter ratio of $T_\sigma = 1:1.5$ between 90% and 10% probability of survival is assumed, the characteristic fatigue strength at $2 \cdot 10^6$ load cycles becomes 35 N/mm^2 for a survival probability $P_s = 97.7\%$.

In the following, the nominal weld stress as well as local stresses are determined for the following geometry, which is close to the average of the first three test series mentioned:

Face plate thickness	t_f	=	8 mm
Web thickness	t_w	=	8 mm
Width of face plate	b_f	=	20 mm (between clamping points)
Height of web	h_w	=	50 mm
Width of the weld	t_{weld}	=	3 mm
Weld eccentricity	e_{weld}	=	0.3 mm

Width of root face left $a_1 = 2.8 \text{ mm}$

Width of root face right $a_2 = 2.2 \text{ mm}$

The nominal stress range is set to $\Delta\sigma_n = 35 \text{ N/mm}^2$, corresponding to the characteristic fatigue strength at $2 \cdot 10^6$ cycles in the tests. A 2D model with plane strain conditions is analysed. Boundary conditions are included in Fig. 4.33.

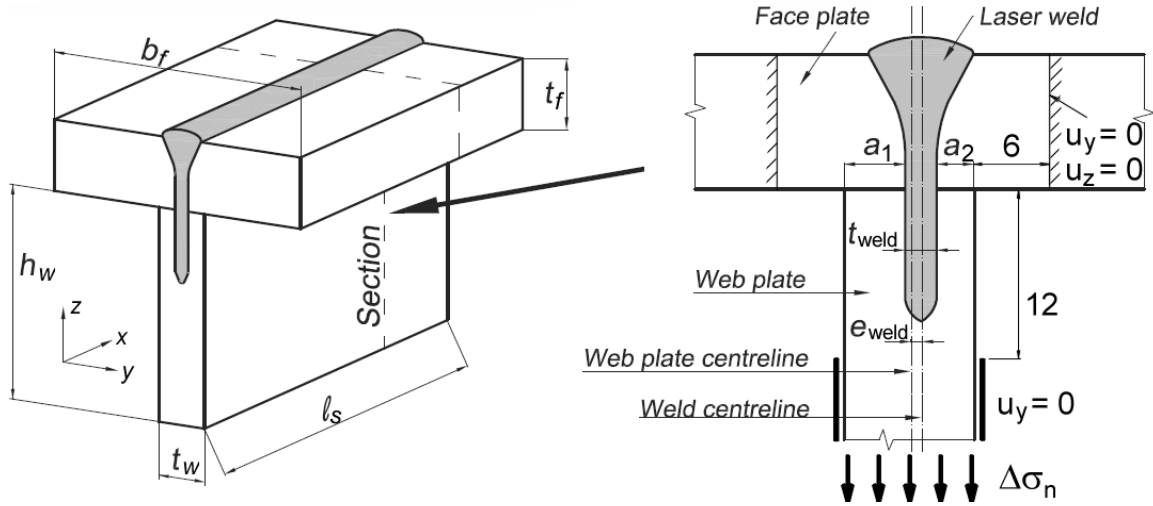


Fig. 4.33: Geometry, loading and boundary conditions of the laser-stake welded T-joint

4.6.2 Application of the nominal stress approach

In the nominal stress approach, the nominal weld stress $\Delta\sigma_{n,w}$ is used. This is simply determined by referring the nominal stress in the web to the weld throat area, resulting in:

$$\Delta\sigma_{n,w} = \Delta\sigma_n \cdot t_w / t_{weld} = 93 \text{ N/mm}^2 \quad (4.5)$$

This value is far above the relevant fatigue class FAT 36, resulting in a fatigue life of 116,000 load cycles if the slope exponent $m = 3$ is used so that the fatigue assessment would be quite conservative compared with the tests.

4.6.3 Application of the effective notch stress approach

The effective notch stress approach with reference radii $r_{ref} = 1 \text{ mm}$ and $r_{ref} = 0.05 \text{ mm}$ has been applied, although the latter is not explicitly recommended for plate thickness of 5 mm and more. However, the weld thickness is rather small. The numerical analysis is performed with two FE-models shown in Fig. 4.34. These models are generated in accordance to the recommendations in chapter 3.4.2 and Fricke (2008) using higher-order 2D elements with eight nodes each. The keyhole notches are positioned in such a way that the weld thickness $t_{weld} = 3 \text{ mm}$ is kept between the edges of the keyhole notches.

The effective notch stress is defined as the maximum principal stress at the edge of the keyhole notches. The right notch with the smaller non-fused root face turned out to have the higher notch stress. Results are given in Table 4.11.

Table 4.11: Computed effective notch stresses, stress concentration factors and fatigue lives for nominal stress $\Delta\sigma_n = 35 \text{ N/mm}^2$ and reference radii $r_{ref} = 1 \text{ mm}$ and $r_{ref} = 0.05 \text{ mm}$

Reference radius r_{ref}	Effective notch stress [N/mm ²]	Stress concentration factor K_f	Fatigue life N for $\Delta\sigma_n = 35 \text{ N/mm}^2$
1 mm	174.8	4.99	3,291,000
0.05 mm	743.8	21.25	938,000

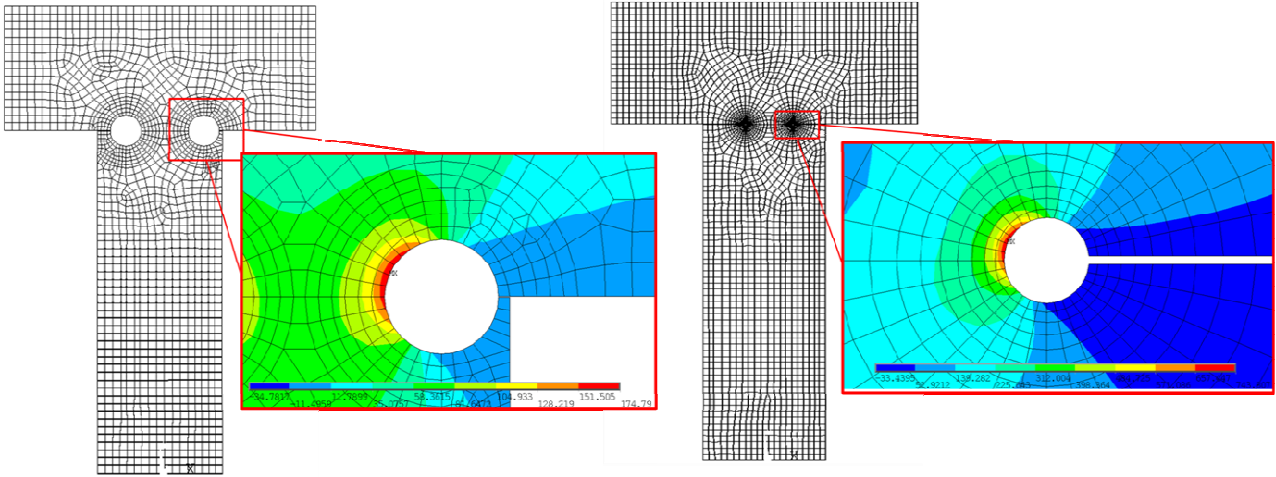


Fig. 4.34: Typical mesh for the effective notch stress analysis with reference radius $r_{\text{ref}} = 1 \text{ mm}$ (left) and $r_{\text{ref}} = 0.05 \text{ mm}$ (right) and max. principal stress distribution

Assuming design S-N curves according to Table 3.1 with FAT 225 for $r_{\text{ref}} = 1 \text{ mm}$ and FAT 630 for $r_{\text{ref}} = 0.05 \text{ mm}$, the computed fatigue lives in Table 4.11 show that the first approach is non-conservative, whereas the latter is conservative at least in the given nominal stress range being the characteristic value of the tests for two million cycles.

4.6.4 Application of the strain energy density (SED) approach

To calculate the SED ΔW another 2-dimensional plane strain FE-model is created which includes two control areas spanned by the radii $R_0 = 0.28 \text{ mm}$ around the notch tips as seen in Fig.4.35.

Within the control areas six elements are arranged along the radius R_0 which results in an element length of $\ell \approx 0.047 \text{ mm}$. The calculated range of the SED in the right notch is $\Delta \bar{W} = 0.0453 \text{ Nmm/mm}^3$ for the nominal stress range $\Delta \sigma_n = 35 \text{ N/mm}^2$. This SED is slightly below the characteristic value given in Fig. 3.14 so that the approach is conservative in this load range.

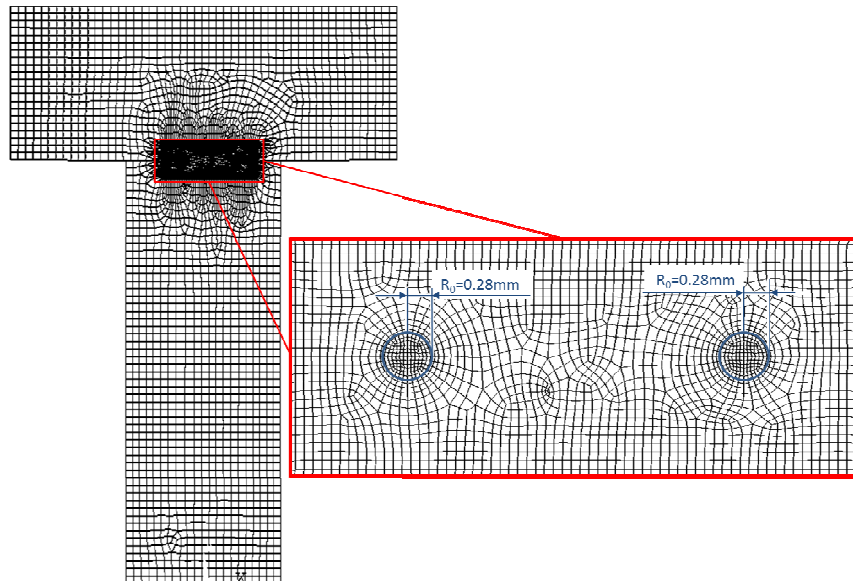


Fig. 4.35: FE-model for the computation of the SED and control radii $R_0 = 0.28 \text{ mm}$ in finer meshed area

Formula (3.16) can be used to obtain the stress intensity factor range ΔK_I . When only mode 1 is observed this equation results in

$$\Delta K_1 = \sqrt{\frac{\Delta \bar{W} \cdot E}{e_1}} \cdot R_0^{1-\lambda_1} \quad (4.6)$$

For the present calculation the notch opening angle is $\alpha = 0$ deg, thus the following parameters are used in accordance to Table 3.2: $e_1 = 0.133$ and eigenvalue of stress field for mode 1 $\lambda_1 = 0.5$. The range of the stress intensity factor becomes $\Delta K_1 = 140.9 \text{ N/mm}^{3/2}$.

4.6.5 Application of the crack propagation approach

The numerical analysis of the stress intensity factors was performed with the program FRANC2D (2004). Fig. 4.36 shows the two-dimensional finite element mesh with eight-node plane strain elements and the finer re-meshed area around the crack path. The initial crack length was set to $a_i = 0.07 \text{ mm}$ (in addition to the non-fused root face $a_2 = 2.2 \text{ mm}$). The final crack length was $a_f = 3.02 \text{ mm}$ corresponding to the weld thickness, see Fig. 4.36.

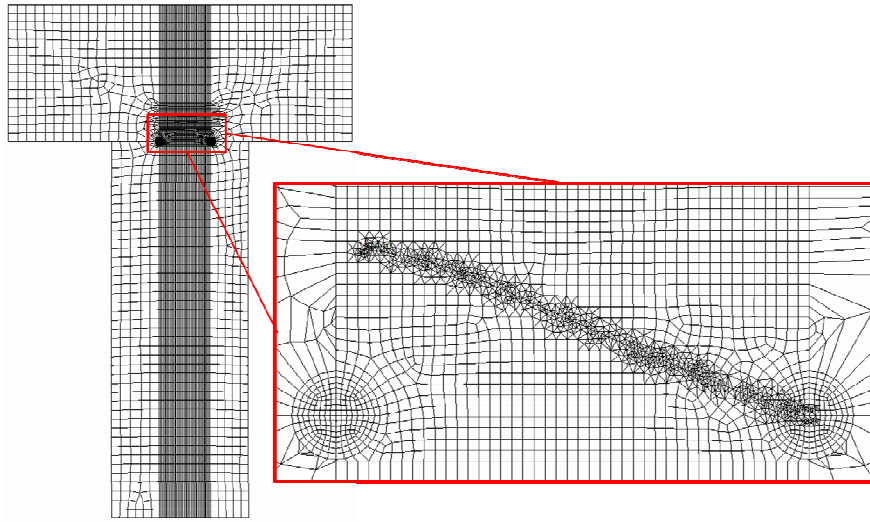


Fig. 4.36: Typical mesh for the crack propagation approach and crack path in refined mesh area

The number of load cycles during crack propagation is estimated by the Paris-Erdogan equation (3.21), again with material parameters $C = 5.21 \cdot 10^{-13} \text{ (N, mm)}$ and $m = 3$ according to Hobbacher (2009). The calculation was performed with a crack length increment $da = 0.05 \text{ mm}$. The final crack length was reached at $N = 0.88 \cdot 10^6$ load cycles which is below the characteristic load cycle number $N_c = 2 \cdot 10^6$. Thus, the crack propagation approach is conservative at least in the given load range.

4.6.6 Comparison of the results

Table 4.12 compares the fatigue lives computed and observed in the tests, based on $P_s = 97.7\%$. The differences are quite large. The crack propagation approach yields the best result.

Table 4.12: Fatigue life N of the laser-stake welded T-joint from different approaches

Analysis method	Test	Nominal stress approach	Effective notch stress approach	Crack propagation approach
No. of load cycles N	2,000,000	116,000	3,290,000	880,000

5 Summary

Weld root fatigue can be a problem particularly for welds without full penetration as the slit between non-welded root faces act like an initial crack.

Several approaches exist for the fatigue assessment of weld roots which have been summarized in the report. In addition to the nominal stress approach which simply utilises an averaged stress in the weld throat, the structural stress approach may be applied e. g. to consider increased stresses or bending in the weld throat. The effective notch stress and the notch stress intensity approaches also consider a single relevant parameter, i. e. a local stress, whereas the stress propagation approach uses the stress intensity factor computed for a growing crack.

The different approaches have been applied to six examples which were partly tested so that a comparison with experiments is possible. The examples include the following details:

- cruciform joints
- Fillet weld around attachment end
- One-sided fillet weld around RHS joint
- Lap joint and cover plates
- Fillet-welded pipe penetration
- Laser-stake welded T-joint

Two to four approaches have been applied to each detail so that the suitability of the approaches for the different details can be assessed.

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7 References

- Andrews R M (1988): The effect of misalignment on the fatigue strength of welded cruciform joints. IIW-Doc. XIII-1180-85, International Institute of Welding.
- Anderson, T.L. (1995): Fracture Mechanics: Fundamentals and Applications. 2nd ed., CRC Press, Boca Raton, USA.
- Anthes R J, Köttgen V B and Seeger T (1993): Kerbformzahlen von Stumpfstößen und Doppel-T-Stößen. *Schweißen und Schneiden* 45(12), 685-688.
- Barsoum Z and Barsoum I. (2009): Residual stress effects on fatigue life of welded structures using LEFM. *Engineering Failure Analysis* 16, 449–467.
- Barsoum Z (2008): Residual stress analysis and fatigue of multi-pass welded tubular structures. *Engineering Failure Analysis* 15, 863–874.
- Boukharouba T; Tamine T; Nui L; Chehimi C and Pluinage G (1995): The use of notch stress intensity factor as a fatigue crack initiation parameter. *Engng Fract Mech* 52, 503-512.
- BS 7608 (1993): Code of practice for fatigue assessment of steel structures. British Standards Institution, London.
- Carpinteri, A., Ed. (1994): Handbook of fatigue crack propagation in metallic structures. Elsevier, Amsterdam.
- DNV (2010): Fatigue Strength Analysis of Offshore Steel Structures. Report DNV-RP-C203, Det Norske Veritas, Høvik.
- Fischer C, Feltz O, Fricke W and Lazzarin P (2011): Application of the Notch Stress Intensity and Crack Propagation Approaches to Weld Toe and Root Fatigue. *Welding in the World* 50, No. 7/8, 30-39.
- Franc2D/L (2004): http://www.cfq.cornell.edu/software/franc2d_casca.htm
- Frank D, Remes H and Romanoff J (2011): Fatigue assessment of laser stake-welded joints. *Int J Fatigue* 33, 102-114.
- Fricke W (2005): Effects of Residual Stresses on the Fatigue Behaviour of Welded Steel Structures. *Mat-wiss u Werkstofftech* 36, pp. 642-649.
- Fricke W, Kahl A and Paetzold H (2006): Fatigue Assessment of Root Cracking of Fillet Welds subject to Throat Bending using the Structural Stress Approach. *Welding in the World* 50, No. 7/8
- Fricke W (2008): Guideline for the Fatigue Assessment by Notch Stress Analysis for Welded Structures. IIW-Doc. XIII-2240r1-08/XV-1289r1-08, Int. Institute of Welding.
- Fricke W and Doerk O (2006): Simplified approach to fatigue strength assessment of fillet-welded attachment ends. *Int J Fatigue* 28, 141-150.
- Fricke W and Kahl A (2006): Fatigue assessment of weld root failure of hollow section joints by structural and notch stress approaches. *Proc of In. Symp on Tubular Structures* (ISTS'11), Quebec.
- Fricke W, Kahl A and Paetzold H (2006): Fatigue assessment of root cracking of fillet welds subject to throat bending using the structural stress approach. *Welding in the World* 50, No. 7/8
- Fricke W and Feltz O (2010): Fatigue tests and numerical analyses of partial-load and full-load carrying fillet welds at cover plates and lap joints. *Welding in the World* 54, No. 7/8, R225-R233.
- Hobbacher A (2009): *Recommendations for Fatigue Design of Welded Joints and Components*. IIW doc.1823-07, Welding Research Council Bulletin 520, New York.
- Hong J K (2010): Evaluation of weld root failure using Batelle structural stress method. Proc. 29th Int Conf on Ocean, Offshore & Arctic Engng (OMAE2010), New York, ASME.

- Lazzarin P and Tovo R (1998): A notch intensity factor approach to the stress intensity of welds. *Fatigue Fract Eng Mater Struct* 21, pp. 1089 - 1103.
- Lazzarin P; Zambardi R (2001): A finite-volume-energy-based approach to predict the static and fatigue behaviour of components with sharp V-shaped notches. *Int J Fracture* 112, 275-298.
- Lazzarin P; Berto F; Gomez F J and Zappalorto M (2008): Some advantages derived from the use of the strain energy density over a control volume in fatigue strength assessments of welded joints. *Int J Fatigue* 30:1345-1357.
- Lazzarin P; Meneghetti G; Berto F and Zappalorto M (2009): Practical application of the N-SIF approach in fatigue strength assessment of welded joints. *Welding in the World* 53, no. 3/4, R76-R89.
- Livieri P, Lazzarin P (2005): Fatigue strength of steel and aluminium welded joints based on generalized stress intensity factors and local strain energy values. *Int J Fract* 133:247-78
- Lotsberg I (2003): Fatigue capacity of fillet welded connections subjected to axial and shear loading. IIW-Doc. XIII-2000-03/XV-1146-03, Int. Inst. of Welding.
- Lotsberg, I. (2004): Fatigue design of welded pipe penetrations in plated structures. *Marine Structures* 17, 29 - 51.
- Maddox, S.J. (1974): Assessing the significance of flaws in welds subject to fatigue. *Welding J., Res. Suppl.* 53:9, pp. 401s - 409s.
- Maddox S J (1991): *Fatigue strength of welded structures*. 2nd Edition, Abington Publishing, Cambridge (UK).
- Maddox S J and Razmjoo G R (1998): Fatigue Performance of Large Girth Welded Steel Tubes. Proc. 17th Int. Conf. Offshore Mechanics and Arctic Engng. (OMAE'98), New York, ASME.
- Maddox S J, Doré M J and Smith S D (2010): Investigation of ultrasonic peening for upgrading a welded steel structure. IIW-Doc. XIII-2326-10.
- Meneghetti G, Lazzarin P (2006): Significance of the Elastic Peak Stress evaluated by FE analyses at the point of singularity of sharp V-notched components, *Fatigue & Fract of Engng Mat & Struct* 30, 95-106
- Meneghetti G (2008): The peak stress method applied to fatigue assessments of steel and aluminium fillet welded joints subjected to mode-I loading, *Fatigue and Fracture of Engineering Materials and Structures*, 31, 5, pp. 346-369.
- Meneghetti G and Lazzarin P (2011): The peak stress method for fatigue strength assessments of welded joints with weld toe and weld root failures. *Welding in the World* 55 (to be published).
- Niemi E, Fricke W and Maddox S J (2006): *Fatigue Analysis of Welded Components - Designer's Guide to the Hot-Spot Stress Approach*. Woodhead Publ., Cambridge.
- Radaj D (1990): *Design and Analysis of Fatigue Resistant Welded Structures*, Cambridge, Abington Publ.
- Radaj D, Sonsino C M and Fricke W (2006): *Fatigue Assessment of Welded Joints by Local Approaches*. Abington: Woodhead Publishing (2nd Edition).
- Radaj D, Lazzarin P and Berto F (2009): Fatigue assessment of welded joints under slit-parallel loading based on strain energy density or notch rounding. *Int J Fatigue* 31, 1490-1504.
- Schijve, J. (2001): *Fatigue of structures and materials*. Kluwer Academic Publ., Dordrecht.
- Sonsino C M (2009a): Effect of residual stresses on the fatigue behaviour of welded joints depending on loading conditions and weld geometry. *Int J Fatigue* 31:1, 88-101.
- Sonsino C M (2009b): A consideration of allowable equivalent stresses for fatigue design of welded joints according to the notch stress concept with reference radii $r_{ref} = 1.00$ and 0.05 mm. *Welding in the World* 53, No. 3/4, R64-R75.

- Sonsino, C M (2009c): Multiaxial Fatigue Assessment of Welded Joints – Recommendations for Design Codes. *Int. J. Fatigue* 31, pp 173-187
- Sonsino, C M (2011): Reliability Aspects in Fatigue Design of Welded Structures Using Selected Local Approaches: the Example of K-Nodes for Offshore Constructions. In: *Fracture and Fatigue of Welded Joints and Structures*, Ed.: K. A. McDonald, Woodhead Publishing, Cambridge, pp. 239-275.
- Sonsino C M, Bruder T and Baumgartner J (2010): S-N lines for welded thin joints - suggested slopes and FAT values for applying the notch stress concept with various reference radii. *Welding in the World* 54, No. 11/12, R375-R392.
- Sonsino C M and Wiebesiek J (2007): Assessment of multiaxial spectrum loading of welded steel and aluminium joints by modified equivalent stress. *IIW-Doc. XIII-2158r1-07/XV-1250r1-07*.
- Sørensen J D, Tychen J, Andersen J U and Brandstrup R D (2005): Fatigue analysis of load-carrying fillet welds. *Proc Int Conf on Offshore Mechanics and Arctic Engng (OMAE'05)*, OMAE-05-1006, New York, ASME.
- Tovo R and Lazzarin P (1999): Relationship between local and structural stress in the evaluation of the weld toe stress distribution. *Int J Fatigue* 21, 1063-1078.
- Turlier D, Klein P and Bérard F (2010): “Seam Sim” method for seam weld structural assessment within a global structure FEA. *Proc. Int. Conf. IIW2010 Istanbul (Turkey)*. AWST, 651-658.
- Turlier D, Klein P, De Noni L, Lawrjaniec D (2011): New FEA shell element model for seam weld static and fatigue structural assessment. *Proc. Int. Conf. Fatigue Design 2011 (CETIM)*.
- Verreman Y and Nie B (1996): Early development of fatigue cracking at manual fillet welds. *Fatigue Fract Eng Mater Struct*, 19, pp. 664 - 681.
- Wiebesiek J and Sonsino C M (2010): New Results in multiaxial fatigue of welded aluminium joints. *IIW-Doc. XIII-2314-10/XV-1349-10*.
- Williams M (1952): Stress singularities resulting from various boundary conditions in angular corners of plates in extension. *J of Applied Mechanics* (ASME) 19, pp. 526 - 528.
- Williams M (1957): On the stress distribution at the base of a stationary crack. *J Appl Mech* 24, 109-114.
- Xiao Z G and Yamada K (2004): Fatigue strength evaluation of root failed joints on one millimeter stress. *J Struct Engng*, 50A:2, 719-726.